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NEW DEVELOPMENTS IN UNDERSTANDING THE HR DIAGRAM

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1. INTRODUCTION

The major goal of stellar evolution theory is the interpretation and reproduction of the Hertzsprung-Russell Diagram (HRD) of stars in different astrophysical environments: solar vicinity, star clusters of the Milky Way and nearby galaxies, fields in external galaxies. The HRD of star clusters, in virtue of the small spread in age and composition of the component stars, is the classical template to which stellar models are compared. If the sample of stars is properly chosen from the point of view of completeness, even the shortest lived evolutionary phases can be tested. The HRD of field stars, those in external galaxies in particular, contains much information on the past star formation history. In this review, no attempt has been made to cover all the topics that could be addressed by a report on the progress made in understanding the HRD. Rather we have selected a

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few topics on which in our opinion most effort has concentrated over the past few years. Among others, the subject of the extension of convective regions in real stars was vividly debated with contrasting appraisals of the problem. Accordingly, various scenarios for the evolution of stars were presented and their far-reaching consequences investigated. Similarly, much effort was dedicated to understanding the evolution of globular cluster stars, and to quantifying the effect of important parameters (see below) in the aim of clarifying whether an age spread is possible. This review presents:

1. a summary of basic stellar evolution theory, whenever possible updated to include the most recent results;
2. a summary of recent results on relevant nuclear reaction rates, radiative opacities, and rates of mass loss by stellar winds;
3. a discussion of several problems related to mixing in stellar interiors (semiconvection and overshoot);
4. a description of the results obtained for the luminosity functions, age, age-metallicity relation, age spread, and *second parameter* of globular clusters;
5. a discussion of the old open clusters as a means for calibrating the extension of convective cores in the range of low-mass stars;
6. similarly for the rich young clusters of the Large Magellanic Cloud (LMC) but in the range of intermediate-mass stars together with the recent progress made in modeling the pulsation of the Cepheid stars and the specific topics of the shape of the instability strip and mass discrepancy;
7. finally, a modern description of the properties of supergiant stars in the Milky Way and LMC together with current understanding of their evolution in light of the problems raised by SN1987A.

For more information the reader is referred to the many review articles that have recently appeared, including Iben & Renzini (1983, 1984), Iben (1985), Castellani (1986), Chiosi & Maeder (1986), Hesser (1988), Renzini & Fusi-Pecci (1988), Iben (1991), VandenBerg (1991), Demarque et al (1991), Fusi-Pecci & Cacciari (1991), and others quoted in the text.

2. BASIC STELLAR EVOLUTION

Independently of the chemical composition, stars can be loosely classified into three categories according to their initial mass, evolutionary history, and final fate: low-mass stars, intermediate-mass stars, and massive stars. Various physical causes concur to define the three groups and related mass limits: 1. the existence of a natural sequence of nuclear burnings from hydrogen to silicon; 2. the amount of energy liberated per gram by gravitational contraction which increases with stellar mass; 3. the tendency of the gas in the central regions to become electron degenerate at increasing density; 4. the existence of threshold values of temperature and density in the center for each nuclear step; 5. the relation between these threshold values and the minimum stellar or, more precisely, core mass at which each nuclear burning can start, and the fact that the minimum core mass for a given nuclear burning is not the same for electron degenerate and nondegenerate gas; finally, 6. the explosive nature of a nuclear burning in a degenerate gas. Because the evolutionary path of a star in the HRD is a natural consequence of the interplay between those physical processes, they will be the main guidelines of our summary of the stellar evolution theory.

2.1 *Low-, Intermediate-, and High-Mass Stars*

By low-mass stars we define those which shortly after leaving the main sequence toward the red giant branch (RGB), develop an electron degenerate core composed of helium. When the mass (M_{He}) of the He core has grown to a critical value ($0.45M_\odot$; the precise value depends weakly on the composition, star mass, and input physics), a He-burning runaway is initiated in the core (He-flash), which continues until electron degeneracy is removed. The maximum initial mass of the star (otherwise called M_{HeF}) for this to occur is about 1.8–$2.2M_\odot$, depending on the initial chemical composition. Stars more massive than M_{HeF} are classified either as intermediate-mass or massive stars. In turn we distinguish the intermediate-mass stars from the massive ones by looking at the stage of carbon ignition in the core. By intermediate mass we mean those stars which, following core He-exhaustion, develop a highly degenerate carbon-oxygen (C-O) core, and as asymptotic giant branch (AGB) stars experience helium shell flashes or thermal pulses. The AGB phase is terminated either by envelope ejection and formation of a white dwarf ($M_{HeF} \leq M_i \leq M_W$) or carbon ignition and deflagration in a highly degenerate core once it has grown to the Chandraseckhar limit of $1.4M_\odot$. The limit mass M_W is regulated by the efficiency of mass loss by stellar wind during the RGB and AGB phases (see Iben & Renzini 1983). This point will be discussed in more detail below. The minimum mass of the C-O core, below which carbon ignition in nondegenerate condition fails and the above scheme holds, is $1.06M_\odot$ corresponding to an initial mass from 7 to $9M_\odot$, depending on the chemical composition. This particular value of the initial mass is known as M_{up}. Finally, massive stars are those that ignite carbon nonviolently and through a series of nuclear burnings proceed either to the construction of an iron core and subsequent photodissociation instability with core collapse and supernova explosion ($M_i \geq M_{mas}$), or following a more complicated scheme undergo core collapse and supernova explosion ($M_{up} \leq M_i \leq M_{mas}$). M_{mas} is about $12M_\odot$.

Figure 1 shows the evolutionary path in the HRD of model stars of $0.8M_\odot$, $5M_\odot$, $20M_\odot$, and $100M_\odot$ which can be considered to be representative of the three categories. These evolutionary tracks possess the chemical composition $Y = 0.250$ and $Z = 0.008$. The thick portions of each track approximately indicate the regions of slow evolution, where the majority of stars are observed. The various evolutionary phases discussed in the text are shown as appropriate for the star mass. The reader should refer to this figure to locate on the HRD the particular evolutionary phase under discussion.

2.2 *Core and Shell H-Burning Phases*

In stars of mass lower than some limiting value M_{con}, the core H-burning on the main sequence occurs in radiative conditions, whereas in stars more massive than this limit the transfer of the liberated energy (nuclear and gravitational) is secured by convection. M_{con} is about $1M_\odot$, depending on the chemical composition.

The core H-burning main sequence phase of stars lighter than M_{con} is characterized by the gradual formation of a small He core at the center and the buildup of a smooth chemical profile from a He-rich core to the outer unprocessed layers. The luminosity steadily increases while the star climbs along the zero-age main sequence itself departing significantly from it toward cooler T_{eff} only at the very end of the phase. The duration of the core H-burning phase strongly decreases with increasing mass of the star going from about 15×10^9 yr for a typical $0.7M_\odot$ star to about 1×10^9 yr for a typical $1.7M_\odot$ star.

After the main sequence phase, the H-exhausted core temporarily cools as electron degeneracy sets in, and the energy liberated by gravitational contraction flows out by electron conduction, delaying the increase in central temperature required to ignite helium in the core. As a low-mass star reaches the base of the RGB, the central temperature reaches a minimum approximately equal to the temperature of the H-burning shell. Thereafter, the mass of the helium core grows under the action of the H-burning shell, the core contracts, and temperatures in the core and H-burning shell increase. The luminosity of the star is proportional to the increase in the shell temperature. The rate at which matter is added to the core by the H-burning shell, and consequently the rate of release of gravitational energy and heating of the core, are proportional to the luminosity. The star climbs the RGB (Hayashi line), while convection in the outer layers gets deeper and deeper, eventually reaching those layers

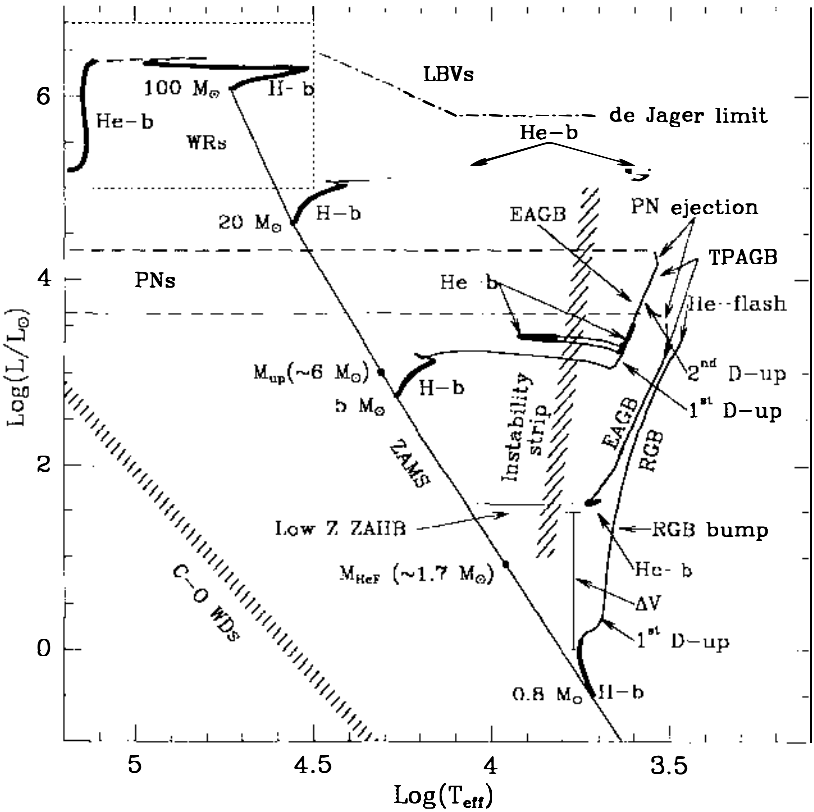

Figure 1 The evolutionary paths in the HRD of model stars of composition $Y = 0.25$ and $Z = 0.008$ and of initial mass $0.8M_\odot$, $5M_\odot$, $20M_\odot$, and $100M_\odot$. The models are calculated with the overshoot scheme for central convection. M_{HeF} and M_{up} are the masses separating low-mass stars from intermediate-mass stars, and the latter from the massive ones, respectively. For low- and intermediate-mass stars the tracks go from the zero-age main sequence (ZAMS) to the end of the asymptotic giant branch (AGB) phase, whereas for the massive stars they reach the stage of C-ignition in the core. Massive stars include the effect of mass loss by stellar wind. H-b and He-b stand for core H- and He-burning, respectively. He-flash indicates the stage of violent ignition of central He-burning in low-mass stars at the tip of the red giant branch (RGB). The main episodes of external mixing (1st and 2nd dredge-up) are indicated by 1st D-up and 2nd D-up, respectively. The AGB phase is separated into the early stages (EAGB) and thermally pulsing regime (TPAGB) of the He-burning shell. For low- and intermediate-mass stars we show the stage of planetary nebula (PN) ejection, the region where PN stars are observed, and the white dwarf (WD) cooling sequence. A horizontal line indicates the locus of the zero-age horizontal branch (ZAHB)—core He-burning models—of low-mass stars with composition typical of globular clusters. The shaded vertical band shows the instability strip of Cepheid and RR Lyrae stars. In the region of massive stars, we show the de Jager limit, the location of the blue luminous variables (LBVs) and Wolf-Rayet stars (WRs). Finally, the thick portions of the tracks indicate the stages of slow evolution, where the majority of stars are observed.

that were nuclearly processed in previous stages and generating a discontinuity in the chemical profile (first dredge-up). The steady outward migration of the H-burning shell forces the external convection to recede. The ascent of the RGB is temporarily slowed when the H-burning shell reaches the discontinuity in the chemical profile. Owing to the electron degeneracy, all low-mass stars, independent of initial mass, build up a helium core of approximately the same mass. When this core has grown to about $0.45 M_{\odot}$, violent He-burning starts off-center because neutrino emission has cooled the innermost regions (Thomas 1967, Mengel & Sweigart 1981). As the nuclear burning progresses inwards, degeneracy is removed, so that a quiescent nuclear burning in the core begins. The RGB phase is terminated. Because this stage occurs at essentially identical core masses, the maximum luminosity of the RGB is almost the same, independent of the initial mass and chemical composition of the star. The duration of the RGB phase depends on stellar mass going from about 2.0×10^9 yr for a typical $0.7 M_{\odot}$ star to about 2.7×10^8 yr for a typical $1.7 M_{\odot}$ star.

The evolution of stars lighter than $M_{\rm HeF}$ but heavier than $M_{\rm con}$ is basically similar to the above scheme, although toward the upper mass end it reflects in many respects the evolution of intermediate-mass stars.

In intermediate- and high-mass stars, the main sequence core H-burning phase is characterized by the formation of a convective core, a steady increase in luminosity and radius, and a decrease of the $T_{\rm eff}$. The size of the convective core, which is customarily fixed by the classical Schwarzschild (1958) criterion ($\nabla_{\rm R} = \nabla_{\rm A}$, with the usual meaning of the symbols), increases with stellar mass, whereas the duration of the core H-burning phase decreases with increasing mass owing to the overwhelming effect of the increasing luminosity. The main sequence core H-burning lifetime goes from several 10^8 yr to a few 10^6 yr as the mass of the star increases from about $2 M_{\odot}$ to $100 M_{\odot}$. Massive stars may be affected by semiconvective instability (thereinafter the H-semiconvection) and mass loss by stellar wind. Semiconvection has long been the characterizing feature of the structure of massive stars evolved at constant mass, whereas to date the most salient signature of the evolution of massive stars is the occurrence of mass loss by stellar wind (see Chiosi & Maeder 1986).

After exhausting central hydrogen while on the main sequence, intermediate- and high-mass stars up to say $15 M_{\odot}$ (the evolution is more complicated for the most massive ones) evolve rapidly to the red giant (supergiant) region, burning hydrogen in a thin shell above a rapidly contracting and heating core, composed essentially of helium. As they approach the Hayashi line, a convective envelope develops whose base extends inward until it reaches layers in which hydrogen has been converted into helium and carbon into nitrogen via the CNO cycle. As a consequence, the surface abundance of those elements varies in a detectable way (first dredge-up). H-burning in the shell not only provides the bulk of the stellar luminosity but also adds matter to the H-exhausted core which continues to grow. When temperature and density in the core reach suitable values, helium is ignited.

The question as to why stars become red giants has been debated for many years without a satisfactory answer. Recent contributions to the subject are by Renzini (1984), Applegate (1988), and Weiss (1989a). Renzini (1984) identifies the physical cause for the rapid expansion of the envelope to red giant dimensions in a thermal instability in the envelope, which is primarily determined by the derivatives of the opacity in the middle temperature region (see also Iben & Renzini 1984). Applegate (1988) finds that a radiative envelope in which a Kramers' opacity law holds cannot transport a luminosity larger than a critical value. He argues that the transition to red giant structure is triggered by the star's luminosity exceeding the critical value. Finally, Weiss (1989a) reanalyzing the criterion introduced by Renzini (1984) concludes that the opacity is not the main cause. The red giant problem still exists.

2.3 *Core He-Burning Phase*

The development of the He-flash at the top of the RGB has been the subject of many quasi-static as well as dynamical studies aimed at understanding whether the violent burning may acquire explosive characteristics or induce some sort of mixing (see the detailed discussion by Iben & Renzini 1984). Arguments exist for excluding both the total disruption of the star (type II-like supernova) and a substantial mixing between the inner core and the outer envelope (see Renzini & Fusi-Pecci 1988). In fact, type II-like supernovae are not seen in elliptical galaxies but post core He-flash stars do exist (HB and AGB), whereas mixing and consequent dredge-up of carbon would produce a kind of red HB star that is not observed. Following He-flash at the termination of the RGB, stars lighter than M_{HeF} quiescently burn helium in a convective core. Their position on the HB depends on several factors, among which the metallicity and the mass of the H-rich envelope dominate, the latter reduced by mass loss from the red giant precursor. For metal-rich stars, the core He-burning is confined to a narrow region or clump near the RGB about 3 magnitudes below the RGB tip, whereas for metal-poor stars the evolution covers a much broader range of T_{eff}s to the blue of the RGB at approximately the same luminosity as for the metal-rich ones. Under favorable circumstances (sufficiently low metal content or high enough mass loss for high metallicity stars) the HB can intersect the instability strip, giving rise to the RR Lyrae pulsators.

The luminosity of the HB stars is determined primarily by the composition (helium abundance) of their main sequence progenitors. The near constant luminosity and duration of the He-burning phase (approximately 10^8 yr) reflect the convergency of precursor stars of different initial mass toward a common value for helium core mass (M_{He}). However, as the contribution to the luminosity by the H-burning shell may depend on the mass of the envelope, blue HB models can be less luminous.

The occurrence of mass loss from the red giant precursors cannot yet be derived from a satisfactory theory, but is basically justified by the observations (e.g. Renzini 1977). The observed morphology of HBs in globular clusters can be successfully explained if the total mass of the HB stars is about 20% smaller than the mass of the progenitors at the tip of the RGB. Furthermore, in order to match the spread in T_{eff}s along an observed HB, it is necessary to suppose that there is a spread in their masses caused by another parameter (probably rotation) which forces the degree of mass loss to vary from star to star (Rood 1973, Renzini 1977). Since the amount of mass lost by a star depends both on the mean mass-loss rate, customarily expressed by empirical relationships as a function of the stellar parameters (e.g. Reimers 1975), and the duration of the phase in which mass loss is supposed to occur, the observational rates and lifetimes along the RGB are such that mass loss plays an important role only in stars with mass smaller than about $1M_{\odot}$, hence typical of old globular clusters (see Iben 1974, Renzini 1977, and Iben & Renzini 1983 for details). All low-mass stars more massive than about $1M_{\odot}$ remain in the clump for the entire core He-burning phase.

In intermediate- and high-mass stars, core He-burning ignites in non-degenerate conditions as soon as the central temperature and density are approximately equal to 10^8 K and 10^4 g cm^{-3}, respectively. This requires a minimum core mass of $0.33M_{\odot}$. Since M_{He} increases with the initial mass of the star because of the larger convective core on the main sequence, the mean luminosity of core He-burning phase increases with stellar mass. During the core He-burning phase, hydrogen continues to burn in a shell at about the same rate as it did during the main sequence phase. The rate at which helium is burnt in the convective core determines the rate at which the star evolves. Typically, the lifetime in the core He-burning stage is about 20 to 30% of the main sequence lifetime, being longer in models of smaller mass.

The slow evolution during core He-burning of intermediate-mass stars takes place in two distinct regions of the HRD, a first near the Hayashi line and a second at higher T_{eff}s and luminosities. The early stages of core He-burning take place in the first region. Subsequently, when the energy released by the burning core (which is increasing) equals the energy released by the H-burning shell (which is decreasing), a rapid contraction of the envelope readjusts the outer layers from convective to radiative and the star moves to the second region, where the remaining part of the core He-burning phase occurs. This causes the blue loops. The precise modeling and lifetime of the second phase depend on the stellar mass, chemical composition, nuclear reaction rates [$^{12}C(\alpha, \gamma)^{16}O$ in particular], extension of the convective core, opacity, mass loss along the RGB, inward pcnetration of the outer convection during the RGB stages (first dredge-up), and other physical details. For any choice of composition, as the stellar mass decreases, the location of the blue loop region moves toward the Hayashi line, eventually merging with the red giant region. Thus, for an assigned chemical composition, core He-burning breaks into two bands, one roughly corresponding to the locus of the Hayashi line or red giant stars, and another that breaks off the red giant band at low luminosities and moves toward the blue with increasing luminosity (the so-called blue band). The mean slope of this band is determined by a complicated interplay among the above physical factors which cannot be established a priori. The blue band of the core He-burning models may intersect the instability strip of the Cepheid stars.

Finally, the location of the core He-burning phase of stars more massive than say $15M_\odot$ is highly uncertain because it is dominated entirely by the effect of mass loss and convective overshoot (see Chiosi & Maeder 1986). These stars will be discussed in more detail below.

The core He-burning phase of intermediate-mass stars (toward the lower mass end) and low-mass stars on the HB is known to be affected by two types of convective instability: in early stages by a semiconvective mixing similar to that encountered by massive stars, and in late stages by the so-called breathing convection. They will be examined in more detail later on. Suffice it to recall that semiconvection prolongs the He-burning lifetime by approximately a factor of two in low-mass stars, i.e. for stars on the HB, whereas it has a negligible effect in intermediate-mass stars. Breathing convection determines a moderate increase in the lifetime (about 20%), whereas it givcs origin to much larger C-O cores in all stars of this mass range.

The result of core He-burning, which turns helium into carbon, oxygen, and traces of heavier species, is the formation of a C-O core whose dimensions are determined by M_{He} and, once again, on the physical model adopted to describe central convection and its efficiency. Low-mass stars form C-O cores of approximately equal mass, whereas all other stars build C-O cores whose sizes increase with stellar mass.

After core He-exhaustion, the structure of the stars is composed of a C-O core, a He-burning shell, and an H-rich envelope at the base of which an H-burning shell is active. However, in massive stars, mass loss by stellar wind may be so strong at this stage that the entire envelope is lost even during the completion of the core He-burning phase (see below).

2.4 *Later Evolutionary Phases*

From the point of view of understanding the HRD, the evolutionary phases beyond the core He-burning of stars more massive than $M_{\rm up}$ are scarcely relevant because of their very short lifetime, hence low probability of detection, were it not for the final supernova explosion. Therefore, their evolution will not be described here (see Chiosi 1986; Woosley 1986, 1988; and Woosley & Weaver 1986).

Following the exhaustion of central helium, low- and intermediate-mass stars evolve through the AGB phase. The AGB phase is separated into two parts: the early AGB or EAGB, which lasts until the H-burning shell is reignited (see below), and the thermally pulsing AGB or TPAGB (see below), which lasts until the H-rich envelope is lost via a normal giant wind (low-mass progenitors) or via a "superwind" (intermediate-mass progenitors). As the abundance of helium in central regions goes to zero, the He-exhausted core contracts and heats up while the H-rich envelope expands and cools. Cooling in the layers external to the C-O core is so effective that the H-burning shell extinguishes. In the HRD the stars evolve running almost parallel to the RGB, and once again the base of the convective envelope penetrates inward. According to Iben & Renzini (1983) there is a limiting mass ($4.6M_{\odot}$ for solar composition) above which external convection eventually reaches layers processed by the CNO cycle. This means that fresh helium and fresh nitrogen are brought to the surface (second dredge-up). Eventually, the expansion of the envelope is halted by its own cooling and the envelope recontracts, the luminosity decreases, and matter at the base of the convective envelope heats up. Ultimately, the H-burning shell is reignited, forcing the envelope convection to move outward in mass ahead of the H-burning shell. This terminates the EAGB. In the meantime, the matter in the C-O core reaches such high densities that the electrons there become degenerate. Electron conduction causes the core to become nearly isothermal, while neutrino cooling carries away the gravitational energy liberated by the material added to the core by the outward migration of the He- and H-burning shells. Therefore, the temperature in the core tends to remain close to the temperature in the He-burning shell (about 10^8 K), well below the threshold value for carbon ignition. Following the reignition of the shell H-burning, nuclear burning in the He-shell becomes thermally unstable (for a more detailed discussion see Iben & Renzini 1983, 1984; and Iben 1991). In brief, the nuclear burning does not occur at a steady rate, but the two shells, one H and the other He, alternate as the major source of energy. For 90% of the time the He-burning shell is inactive and the H-burning shell is the major source of energy. However, as the mass of the He-rich zone below the H-burning shell increases, the density and temperature at the base of this zone increase until the rate of energy production by the $3\alpha - {}^{12}C$ reaction becomes higher than the rate at which energy can be carried out by radiative diffusion. As originally discovered by Schwarzschild & Harm (1965), a thermonuclear runaway occurs. A thin convective layer is generated on the top of the He-burning shell. At first the energy goes into raising the temperature of and expanding the matter in and near the burning zone and the material is pushed away in both directions. Matter at the base of the H-burning region is pushed out and cools to such low temperatures that the H-burning shell is temporarily extinguished. Eventually, matter at the He-burning region begins to cool as it overexpands and the rate of burning there drops dramatically. The convective layer disappears and a steady state is reached in which He-burning occurs quiescently at a slowly decreasing rate as the He-burning shell actually runs out of fuel. This quiescent phase lasts for about 10% of the time elapsing between successive outbursts. The material propelled outward falls back and the H-burning shell eventually reignites. During this phase, material processed into the intershell region can be brought into the outer convective envelope and exposed to the surface. The so-called third dredge-up can then take place. In AGB stars of large C-O core mass (hence with large initial mass) the dredge-up can occur easily. But in AGB stars of small C-O core mass (hence with small initial mass) this is possible only if extra mixing is forced into the intershell region. The goal is achieved either by means of semiconvection induced by the more opaque C-rich material deposited in the intershell region by the tiny convective shell ahead of the flashing He-burning shell or by crude overshoot of convective elements from the convective shell itself. The mechanism of semiconvection has been proposed by Iben & Renzini (1982) following a suggestion by Sackman (1980). Convective overshoot has subsequently been used by Hollowell (1988) and Hollowell & Iben (1988, 1989). In both discussions C-rich material is deposited in more external layers where it can be easily engulfed by the external convection during the subsequent cycle. This is the basic mechanism to convert an M giant into a carbon star (C star).

Along the TPAGB phase, the luminosity of the star increases linearly with the mass of the H-exhausted core (Paczynski 1970a,b) and the star brightens in $M_{\rm bol}$ at a constant rate (see Renzini 1977 and Iben & Renzini 1983 for details).

Given that C-ignition in highly degenerate conditions requires a C-O core mass of $1.4 M_{\odot}$ (the Chandrasekhar limit), and considering the effect of mass loss, the minimum initial mass of the star, $M_{\rm W}$, for C-ignition to occur is estimated in the range 4 to $6M_{\odot}$, depending on the adopted mass-loss rates, evolutionary lifetimes, and chemical composition (Iben & Renzini 1983). Stars lighter than the above limit will fail C-ignition and, by losing the H-rich envelope will become C-O white dwarfs (WDs) with a modest increase of the C-O core mass during the TPAGB phase (about $0.1M_{\odot}$). In a very low-mass star $(0.8\text{–}1.0)M_{\odot}$, the ejection of the envelope may be completed even before the H-burning shell is reignited and the thermal pulsing regime begins. Direct observational evidence for the existence of the TPAGB phase and third dredge-up is given by properties of long-period variables (LPVs) with enhanced strength of the ZrO band. In fact, Zr is formed by s-processing in the convective He-burning shell during a shell flash and is dredged up to the surface (Wood et al 1983).

However, even if intermediate-mass stars with initial mass in the range $M_W \leq M \leq M_{up}$ could experience deflagrating C-ignition, in actuality they do not for several reasons (Iben 1985, 1991). In short, as we infer from the density of matter in planetary nebula (PN) shells, the estimated outflow rates from OH/IR sources, the several nearby C stars, the paucity of C stars in rich clusters of the LMC (like NGC 1866), and finally the luminosity function of carbon stars in the same galaxy (Reid et al 1990), there must be some fast mechanism, which on a very short time scale (10^3yr) terminates the TPAGB phase soon after it begins with a modest increase in the mass of the C-O core with respect to the initial value. The sudden termination of the AGB phase of all intermediate-mass stars has been long attributed to a sort of "superwind" (see Renzini & Voli 1981, and Iben & Renzini 1984), the physical interpretation of which is not yet understood. The manifestation of the superwind could be the OH/IR phenomenon. Estimates of the mass-loss rates from AGB stars and speculations about the physical nature of the superwind have been made by many authors among whom we recall Baud & Habing (1983), Bedijn (1988), van der Veen (1989), and Bowen & Willson (1991). Computations of AGB models including the effect of mass loss are still rare. Recent calculations are by Wood & Vassiliadis (1991) who identify the superwind in the combined effect of large amplitude radial pulsations and radiation pressure on grains.

The arguments presented above also suggest that the maximum mass of WDs is $1.1M_{\odot}$, considerably lower than the Chandrasekhar mass of $1.4M_{\odot}$, and slightly larger than the value of the C-O core mass at the start of the TPAGB for a star with initial mass equal to M_{up} (see Iben 1991 and Weidemann 1990).

2.5 *Carbon Stars*

Stars on the AGB are classified as either oxygen-rich objects (M-type stars) or carbon-rich objects (C-type stars) based on the abundance ratio of carbon and oxygen atoms. In the current understanding of these stars, the third dredge-up mechanism gradually converts an M star into a C star. Since the early studies on carbon stars and other AGB stars in our Galaxy and Magellanic Clouds (Blanco et al 1978, 1980) it has become evident that although the theoretical AGB star distributions (Iben 1981, Renzini & Voli 1981) based on classical models of Iben (1975a,b; 1976) predict that C stars can only form at high luminosities (no stars are expected less luminous than $M_{\rm bol} = -5$ or brighter than $M_{\rm bol} = -7.5$), the observations indicate that C stars occur in the range $-3.5 \leq M_{\rm bol} \leq -6$. This topic has been widely discussed by Iben (1988) and Lattanzio (1988a); the most recent revision of the observational data based on Magellanic Cloud studies is by Reid et al (1990, and references therein). The series of recent models by Lattanzio (1986, 1987a,b, 1988a,b, 1989, 1991), Boothroyd & Sackman (1988a,b,c,d), and Hollowell (1987, 1988) in the mass range 1–$3M_{\odot}$, in which different algorithms for treating convection and modern opacities have been used, can produce models of carbon stars of quite low luminosity in better agreement with the observed luminosity function of these stars.

However, the theoretical scenario on which rests the explanation of the luminosity function of C stars has been changed recently by Bloecher & Schoenberner (1991). They have shown that the concept of the core mass-luminosity relationship (Paczynski 1970a,b) breaks down if outer convection is so efficient that envelope burning can occur (Renzini & Voli 1981). Instead the models evolve rapidly to much higher luminosities without a corresponding rapid increase in the core mass. This means that an a priori limit to the luminosity attainable by an AGB star does not exist with possible implications for the ages of star clusters derived from the luminosity of their C stars (Mould & Aaronson 1982, Iben & Renzini 1983). Furthermore, the fast increase in the luminosity and the likely enhancement in the mass-loss rate could explain the decline (extreme paucity of stars) in the luminosity function of bright AGB stars. In addition to this, the advent of models with convective overshoot has lowered the value of $M_{\rm up}$ (see below) to about $5M_{\odot}$ instead of 8–$9M_{\odot}$, thus subtracting from the family of C star progenitors those with the highest initial luminosity (Bertelli et al 1985, Chiosi et al 1986). The Bloecher & Schoenberner (1991) result is not in conflict with the evolutionary scheme based on models with convective overshoot.

As for the initial mass of the C star progenitors, according to Bryan et al (1990), the observed luminosity of AGB stars indicates that these are smaller than $3M_{\odot}$. Claussen et al (1987) deduced a mass range for the progenitors of C stars between 1.2–$1.6M_{\odot}$. On the basis of kinematical properties, Barnbaum et al (1991) consider the existence of a population of C stars that is more massive than that described by Claussen et al (1987). The initial mass of the C stars of this group should be above 2.5–4$M_\odot$.

Finally, the discovery of Azzopardi et al (1985, 1988) of a group of C stars in the Galactic Bulge has rendered the understanding of these objects even more puzzling. Specifically, the bulge C stars are bluer and intrinsically much fainter compared to most other known C stars. They are also likely to be metal-rich and possibly super metal-rich (Azzopardi et al 1988). Once again, these C stars run counter to current models. In fact, the recent calculations by Lattanzio (1989) show that models resembling C stars have either low mass (1$M_\odot$) and low metallicity ($Z = 0.001$) or larger mass (1.5$M_\odot$) and larger (solar) metallicity. According to Westerlund et al (1991) the stars in the Galactic Bulge should be old, possess a mass of about 0.8$M_\odot$, and have a metallicity in the range 0.1 solar to a few times solar (Rich 1988). The origin of the bulge C stars is therefore a mystery (Lequeux 1990).

2.6 *Planetary Nebulae and White Dwarfs*

The main parameters governing the evolution from the AGB to the WD stage through the planetary nebula (PN) phase are the precise stage in a thermal cycle at which the final ejection of the H-rich envelope occurs and the amount of H-rich material which is left on the surface of the remnant at the termination of the ejection phase. Summarizing the results of many authors (Schoenberner 1979, 1981, 1983, 1987; Iben 1984, 1989; Iben & MacDonald 1985, 1986; Iben et al 1983; Renzini 1979, 1982; Wood & Faulkner 1986), three evolutionary schemes are possible: 1. the ejection of the envelope occurs during the quiescent H-burning interpulse phase and the mass $dM_{\rm He}$ of the helium layer between the C-O core and the bottom of the H-rich envelope is "small," i.e. in the range 0.2 to 0.8 $dM_{\rm H}$, where $dM_{\rm H}$ is the mass processed by the H-burning shell between He-shell flashes on the AGB (for a 0.6$M_\odot$ core, $dM_{\rm H}$ is about 0.01$M_\odot$); 2. the ejection occurs in the same stage but $dM_{\rm He}$ is greater than 0.8$dM_{\rm H}$; or 3. the ejection of the H-rich envelope occurs during a He-shell flash or shortly thereafter.

In the first case, following the loss of the envelope, the star evolves blueward at about constant luminosity sustained by the H-burning shell at the base of the residual envelope. The surface temperature gets higher at decreasing mass of the H-rich surface layer. At $T_{\rm eff} \geq 30{,}000$ K, the flux emitted by the central star ionizes the surrounding nebula and the complex appears as a PN. The time scale is of the order of 10^4 yr for a 0.6$M_\odot$ star. In this phase, stellar winds from the central stars are known to occur at rates of about 10^{-9}–$10^{-7}M_\odot$/yr (Perinotto 1983, Cerruti-Sola & Perinotto 1985). When the mass of the H-rich surface layer falls below $10^{-2}dM_{\rm H}$, the H-burning shell extinguishes. The surface layers contract and the luminosity—which is a complicated consequence of gravitational energy release, cooling of nondegenerate ions, and neutrino losses (e.g. D'Antona & Mazzitelli 1990)—drops dramatically. Gravitational diffusion becomes so strong that heavy elements sink and hydrogen, if any is left, becomes the dominant element at the surface. Ultimately, the star settles onto the cooling sequence of WDs for the given mass and composition. This model approximates well the characteristics of observed DA-WDs.

In the second case, a final He-shell flash is possible. Following the extinction of the H-burning shell as in the previous case, helium ignites in a shell and the star is pushed back to the tip of the AGB. There the same mechanism that removed the H-rich envelope when the star left the AGB for the first time is likely to operate for a second time, forcing the departure from the AGB. However, in this case the luminosity of the star is sustained by the He-burning shell and departure from the AGB requires that mass loss continues until the residual mass of the H-rich material is less than $10^{-5}M_{\odot}$. Evolving to high T_{eff} once again the PN is reexcited, and stellar winds from the central stars cause the loss of all remaining H-rich matter at the surface. The duration of this phase is about three times longer than in the previous case. Eventually He-burning ceases and gravitational sinking of heavy elements makes helium the dominant element at the surface. Finally, the star settles onto the WD sequence. This model nicely corresponds to non-DA-WDs.

In the third case, the H-rich envelope is ejected during a He-shell flash when the intershell region contains the smallest amount of mass. Departed from the AGB, the luminosity, sustained by the He-burning shell, fades to a minimum as the star evolves to higher T_{eff}s. At a certain point, hydrogen is reignited and the luminosity increases again at almost constant T_{eff}. The subsequent evolutionary track lies close to the corresponding H-burning track (Iben 1984, Wood & Faulkner 1986). In coincidence with H-reignition, a marked slowdown of the evolutionary rate occurs. AGB stars becoming WDs through this scheme, after about 10^7 yr from the extinction of the He-burning shell for a typical $0.6M_{\odot}$ star, may undergo a final H-burning runaway leading to an outburst which exhibits the characteristics of a slow nova (Iben & MacDonald 1985, 1986). There are some reasons to believe that planetary nebulae are the descendents of AGB stars via the ejection of the envelope during helium shell flashes (Iben 1991, Dopita et al 1991). Whether the slow nova episode is a common feature has not yet been tested.

The structure and evolution of WDs has been reviewed by D'Antona & Mazzitelli (1990), whereas the problems with the masses, mass distribution, and evolutionary status of these stars have been described by Weidemann (1990), to whom the reader should refer for further details. The distribution of single WDs with respect to the mass is based upon the position in the HRD and mass-radius relationship. The distribution is strongly peaked at $M = 0.55$–$0.60 M_{\odot}$, it extends downward at $0.4 M_{\odot}$, and falls off exponentially beyond $M = 0.6 M_{\odot}$ (Iben 1991, Weidemann 1990).

3. NUCLEAR REACTIONS, OPACITIES, AND STELLAR WINDS

3.1 *Nuclear Reactions*

Although the rates of the major reactions involved in H and He-burnings are sufficiently known, the works of Fowler et al (1967, 1975) and Harris et al (1983) show that many of them have changed over the years and that some uncertainty is still possible. This is particularly true for the $^{12}C(\alpha, \gamma)^{16}O$ reaction whose cross section has been varied several times. The measurements by Kettner et al (1982) and the analysis of Langanke & Koonin (1982) increased the cross section by a large factor (from 3 to 5) with respect to the Fowler et al (1975) estimate. Subsequent revision of the rate (Fowler 1984, Caughlan et al 1985) set the increase at about 2–3 times the old value. The most recent revision of this problem (Caughlan & Fowler 1988) has lowered the rate to nearly the same value as in Fowler et al (1975). The effects of varying the rate of this reaction on evolutionary models have been known since the early study by Iben (1972). Specifically, the larger the cross section, the greater is the extent to which carbon is converted into oxygen, and the further the loop extends to the blue before rapid core contraction and envelope expansion set in and the evolution proceeds back to the red. The core He-burning lifetime is increased by a few percent. In massive stars, however, the above effects are blurred by mass loss and convective overshoot (see below). However, the abundance of carbon can get so low that the C-burning phase is actually missing, with profound consequences on later evolution of these stars (e.g. Woosley 1986, 1988; Woosley & Weaver 1986).

3.2 *Opacities*

For more than twenty years, almost all evolutionary calculations were made with the Los Alamos Opacity Libraries (LAOL) based on the work of many groups (Cox & Stewart 1965, 1970a,b; Cox & Tabor 1976; Magee et al 1975; Huebner et al 1977; Weiss et al 1990). Occasionally, other opacities calculated by T. R. Carson (1976, unpublished) were used (Carson & Stothers 1976, 1988; Carson et al 1981; Stothers 1976; Stothers & Chin 1977, 1978). The role played by each particular source of opacity in building up the total radiative opacity has been discussed by Iben & Renzini (1984), to whom we refer. The region in the temperature range

5×10^5 to 5×10^6 K, where the bound-bound and bound-free transitions of elements from carbon to iron dominate the opacity, is the one suffering from the highest uncertainty. The high number of electrons in each elemental species, the large number of ionization and excitation stages, the non-hydrogenic structure of the electronic configuration, and the distortions of this induced by nearby electrons and ions that are difficult to model, all conspire to make the opacity in this temperature range very difficult to calculate, and therefore subject to continuous upgrading. This is the main reason why the characteristics of stellar models that are very sensitive to the so-called middle temperature opacity are still highly uncertain and a matter of debate. LAOL opacities were based on the hydrogenic atomic model, whereas Carson's opacities stood on the hot Thomas-Fermi approximation. The two opacities were quite similar except for the region of the CNO ionization where in the Carson opacity a pronounced bump was present. Although Carson's opacities were retracted by Carson et al (1984), various reasons suggested that an increase of this type was indeed necessary. For example the possibility that opacity enhancements could be responsible for the pulsation of β-Cephei stars was examined by Stellingwerf (1978). Simon (1982) noticed that by increasing the opacity of the metals by factors of 2–3 for $10^5\ \mathrm{K} \leq T \leq 10^6$ K in models of classical Cepheids, the observed period ratios could be reproduced for masses and luminosities in agreement with those of evolutionary models. Bertelli et al (1984) introduced an opacity bump in the CNO ionization region of the LAOL opacity and studied the effects on the location in the HRD of models with core overshoot and mass loss. The opacity peak was set at $\log T = 5.80$ and the opacity was increased by a factor of 2–3. These models were particularly successful in explaining the overall properties (main sequence extension, lifetimes, etc) of massive stars. The suggestion by Simon (1982) was confuted by Magee et al (1984) who claimed that such opacity increase was incompatible with atomic physics. However, the opposite conclusion was reached by Iglesias et al (1987, 1990) and Iglesias & Rogers (1991a,b) who presented new opacity calculations using improved atomic physics showing that a significant increase in the opacity (bump-like structure) is present at about $\log T = 5.38$. Using the opacity calculations of Iglesias et al (1987) as a guide, Andreasen & Petersen (1988), Andreasen (1988), and Petersen (1989, 1990) artificially enhanced the LAOL opacity by factor α in the range $1.5\times10^5 \leq T \leq 8\times10^5$ K. Adopting $\alpha = 2.5$ they reproduced the period ratios for double mode Cepheids of population I, whereas using $\alpha = 1.5$ they resolved the mass anomalies for the lower metallicity RR Lyrae stars. The enhancement and mass range proposed for the population I Cepheids is in close agreement with the suggestion of Bertelli et al (1984) for supergiant stars. Those suggestions for an opacity enhancement are confirmed by the recent opacity calculations of Iglesias & Rogers (1991a,b) and Rogers & Iglesias (1991). Systematic studies of opacity are underway by at least two independent groups, i.e. the OPAL project at the Livermore Laboratory (Iglesias & Rogers 1991a,b; Rogers & Iglesias 1991) and the OP project (Seaton 1987, 1991; Berrington et al 1987). Evaluations of the effects of the new opacities on stellar models is at work in various groups (Bressan et al 1991, Stothers & Chin 1991) and any conclusion is premature.

3.3 *Stellar Winds*

In recent years, quantitative mass-loss theory applicable to massive early type stars has been developed (Castor et al 1975, Abbott 1982, Pauldrach et al 1986, Owocki et al 1988) which allows one to calculate the mass-loss for a given star. Self-consistent models in which the mass-loss rate results from the physical properties of the stars rather than an a priori assumption are not yet available even though efforts have been made in this direction (Meynet 1991). No satisfactory stellar wind models are available for yellow and red supergiants (see Lafon & Berruyer 1991 for a recent review of the subject). The most recent compilation and parameterization of the mass-loss rates of galactic stars all over the HRD is by de Jager et al (1988), which holds for stars from the main sequence to the latest spectral types. Suitable dependencies on the metallicity are often included to account for the fact that the mass-loss rates are expected to decrease with metallicity (see Maeder 1990). For the stars above the so-called de Jager limit (LBV candidates) the rate of mass loss is customarily increased to about $10^{-3} M_{\odot}/\mathrm{yr}$, whereas for the WR stars the formulation by Langer (1989a,b) is adopted. It is worth recalling that according to current empirical mass-loss rates, massive stars lose much more mass than is expected from the theoretical mass-loss rates, in particular during the core H-burning phase. The reason for the discrepancy is not known.

The problems related to detection and modeling of mass loss in evolved cool stars have been reviewed by Lafon & Berruyer (1991). [See also Skinner & Whitemore (1988) and van der Veen & Rugers (1989) for the existence and use of an empirical correlation between the IR colors of the stars and the mean mass-loss rates derived from CO data.] As for the mechanism powering the wind from late type stars, the high mass-loss rates observed in Mira stars, which are in excess of $10^{-8} M_{\odot}/\mathrm{yr}$ and often as large as $10^{-4} M_{\odot}/\mathrm{yr}$, together with the high luminosity of these stars suggested that the mass-loss mechanism is the radiation pressure on the gas. However, standard opacities of the atmospheric layers were too small to effectively accelerate the gas against gravity. The inclusion of H_2O to the opacity (Alexander et al 1989) could alleviate the problem (Elitzur et al 1989). Subsequently, the key mechanism to transfer momentum from radiation field to gas was identified in the opacity due to graphite (in carbon stars) and dust grains formed by coagulation of oxides and carbides of heavier elements (Si, Mg). Since the opacity of dust is very high, the radiation force on dust can overcome gravity and the momentum of the dust is imparted to the gas which is dragged along. This mechanism was first suggested by Kwok (1975) and has been applied to AGB stars by many authors (Gail & Sedlmayr 1987 and references therein). However, the main problem with this model is that dust forms at large distances from the star, where the density is too low to build a significant wind. Therefore, it was suggested that another mechanism should exist extending the stellar atmospheres and increasing the density at radii where dust can form. Bowen (1988) has shown that stellar pulsation can enhance the atmospheric density scale height and can drive a wind together with radiation forces on dust. The connection between pulsation and mass loss has been thoroughly discussed by Willson (1988) to whom we refer. Often, useful dependencies of the mass-loss rates on basic stellar parameters are given (see for instance Volk & Kwok 1988). This mechanism cannot be applied to those giants showing substantial mass loss and no evidence of circumstellar dust. To this aim, another mechanism was advanced, in which sound waves are responsible for the mass loss. The sound waves are generated either by convection in the mantle of the star or by pulsation at high eigenmodes. Models of this type were applied to AGB stars by Pijpers (1990), Pijpers & Hearn (1989), and Pijpers & Habing (1989).

Finally, there has been much speculation about the nature and cause of the fast mass loss otherwise called superwind (e.g. Iben & Renzini 1983, Iben 1987). Recent work on the subject is by Bowen & Willson (1991) who calculated large grids of dynamical atmosphere models for Mira-like stars. They found that as a natural consequence of evolutionary changes in stellar parameters, the mass-loss rate increases as an approximately exponential function of time and the final evolution is characterized by a powerful wind that resembles the kind of superwind first advocated by Renzini & Voli (1981).

4. SEMICONVECTION AND CONVECTIVE OVERSHOOT

4.1 *Hydrogen Semiconvection*

During the core H-burning phase of massive stars on the main sequence, radiation pressure and electron scattering opacity give rise to a large convective core surrounded by an H-rich region, which is potentially unstable to convection if the original gradient in chemical abundance is maintained, but stable if suitable mixing is allowed to take place. Theoretical models picture this region undergoing sufficient mixing until the condition of neutrality is restored, but carrying negligible energy flux. The gradient in chemical abundance depends on which condition is used to achieve neutrality, either Schwarzschild (1958) or Ledoux (1947). The former condition tends to give smoother chemical profiles and in some cases leads to the onset of a fully intermediate convective layer. It is worth recalling that the Ledoux criterion is a stronger condition favoring stability with respect to the Schwarzschild criterion. Similar instability occurs also during the early shell H-burning stages. The effects of H-semiconvection on the evolution of massive stars have been summarized by Chiosi & Maeder (1986) and most recently by Chiosi et al (1991).

4.2 *Helium Semiconvection*

As He-burning proceeds in the convective core of stars of any mass, the C-rich mixture inside the core becomes more opaque than the C-poor material outside; therefore the radiative temperature gradient increases within the core. The resulting superadiabaticity at the edge of the core leads to a progressive increase (local convective overshoot) in the size of the convective core during the early stages of He-burning (Schwarzschild 1970, Paczynski 1971, Castellani et al 1971a,b). Once the convective core exceeds a certain size, the continued overshooting is no longer able to restore the neutrality condition at the border due to a characteristic turn-up of the radiative gradient. The core splits into an inner convective core and an outer convective shell. As further helium is captured by the convective shell, this latter tends to become stable, leaving behind a region of varying composition in which $\nabla_R = \nabla_A$. This type of mixing is called semiconvection. The extension of the semiconvective region varies with the star mass, being important in low- and intermediate-mass stars up to say $5M_\odot$, and negligible in more massive stars. Various algorithms have been devised to treat semiconvection (Castellani et al 1971b; Demarque & Mengel 1972; Sweigart & Demarque 1972; Gingold 1976; Robertson & Faulkner 1972; Sweigart & Gross 1976, 1978; Castellani et al 1985; Lattanzio 1986, 1987b, 1991; Fagotto 1990). In all computed models, when $Y_c \leq 0.1$, the convective core may undergo recurrent episodes of rapid increase followed by an equally rapid decrease until it engulfs the whole semiconvective region. Castellani et al (1985) have designated this phase as "breathing pulses of convection." Semiconvection increases the core He-burning lifetime (by approximately a factor of two), whereas breathing convection increases the mass of the C-O core leftover at the end of He-burning phase. This fact will greatly shorten the early AGB phase. The only exception to this scheme are models calculated by Gingold (1976) in which for some chemical compositions the breathing convection phase is apparently missing. The reason for its absence has never been understood. Models with semiconvection and models with semiconvection plus breathing convection have different predictable effects on the expected ratio of the number of AGB stars to the number of HB stars in well studied globular clusters. Renzini & Fusi-Pecci (1988), comparing the above ratio with Gingold's (1976) models, consider semiconvection as a true theoretical prediction and argue that breathing convection is most likely an artifact of the idealized algorithm used in describing mixing (see also Chiosi 1986). Given that breathing convection is a consequence of the time-independent treatment of semiconvection, and that both are based on local descriptions of mixing, the question arises whether nonlocal, e.g. full convective overshoot (see below), and/or time-dependent mixing may overcome the above difficulties.

4.3 *Convective Overshoot*

The argument for the occurrence of convective overshoot is that the traditional criteria for convective stability focus on the locus where the buoyancy acceleration vanishes. Since it is very plausible that the velocity of the convective elements is not zero at that layer, these will penetrate (overshoot) into regions that are formally stable. If the physical ground of convective overshoot is simple, its formulation and efficiency are much more uncertain. This uncertainty is reflected in the variety of solutions and evolutionary models that have been proposed over the years. Major contributions to this subject are from Shaviv & Salpeter (1973), Maeder (1975), Cloutman & Whitaker (1980), Bressan et al (1981), Stothers & Chin (1981, 1990), Matraka et al (1982), Doom (1982a,b; 1985), Bertelli et al (1985, 1986a,b), Bressan et al (1986), Xiong (1983, 1986), Langer (1986), Baker & Kuhfuss (1987), Renzini (1987), Maeder & Meynet (1987, 1988, 1989, 1991), Aparicio et al (1990), Alongi et al (1991a,b), and Maeder (1990). In those studies the overshoot distance at the edge of the convective core has been proposed between zero and about $2H_p$ (pressure scale height). As many evolutionary results depend on the extension of the convective regions, this uncertainty is most critical. Because a generally accepted theory for overshoot is not yet available, most of those studies sought to constrain the efficiency of overshoot from the convective core by comparing parameterized models with observations. Maeder & Mermilliod (1981) analyzing clusters like the Pleiades noticed that the main sequence extends to too bright a luminosity to fit standard models (Brunish & Truran 1982a,b), and suggested a certain amount of overshoot. Mazzei & Pigatto (1989) showed that the sequential star formation invoked by Stothers (1985) to fit the Pleiades is not necessary if overshoot models are adopted. Barbaro & Pigatto (1984) and Chiosi & Pigatto (1986) argued for overshoot in stars with masses in the range 1.5–2.2$M_\odot$ by pointing out that, although the base of the RGB is populated in clusters older than about 2–3 × 10^9 yr, it is not well populated in clusters with age 1–2 × 10^9 yr, as if in this mass range degenerate He-ignition and He-flash were avoided in contrast with classical models. Barbaro & Pigatto (1984) and Bertelli et al (1985) suggested that overshoot could lead to larger core masses and hence to nondegenerate core He-ignition also in this range of stellar ages (initial masses). Convective overshoot was invoked by Andersen et al (1990) and Napiwotzki et al (1991) to explain the position in the HRD of a few stars with well determined $T_{\rm eff}$s and gravities. Aparicio et al (1990) and Bertelli et al (1992) studying old open clusters, like King 2 and IC 4651, whose turnoff mass is in the range 1.5 to 2$M_\odot$, concluded that a certain amount of convective overshoot was necessary. Following the early study by Becker & Mathews (1983), Chiosi et al (1989a,b) examined the key LMC cluster NGC 1866, where the turnoff mass is 4–5$M_\odot$, and convincingly showed that overshoot models are a better fit to both the overall morphology of the HRD and the luminosity function of main sequence stars. This conclusion was also reinforced by the study of the Cepheid stars in the LMC cluster NGC 2157 by Chiosi et al (1992a), where it was shown that the use of overshoot models brings into agreement the evolutionary and pulsational mass of these stars (see below). Finally, the need for convective overshoot in young galactic clusters was discussed in great detail by Mermilliod & Maeder (1986) and Maeder & Meynet (1987, 1988, 1989) to whom we refer.

In addition to the convective core, overshoot may occur at the bottom of the convective envelope during the various phases in which this develops, such as on the RGB. The effect of envelope overshoot on stellar models of low- and intermediate-mass stars has been studied by Alongi et al (1991a), whereas that for high-mass stars by Chiosi et al (1991).

4.4 *Stellar Models with Convective Overshoot*

The core H-burning phase of all stars possessing a convective core on the zero-age main sequence ($M \geq M_{\rm con}$) is affected by convective overshoot. Because of the larger cores, the models run at higher luminosities and live longer than the classical ones. They also extend the main sequence band over a wider range of $T_{\rm eff}$s, this trend increasing with stellar mass (e.g. Alongi et al 1991a,b; Bertelli et al 1985, 1986a,b; Maeder & Meynet 1987, 1988, 1989, 1991). Massive stars ($M \geq 40M_\odot$) would spread all across the HRD, were it not for the contrasting effect of mass loss (sce Bressan et al 1981). The mass range $M_{\rm con} \leq M \leq 2M_\odot$, where the onset of the convective core takes place gradually, deserves particular attention because the time scale required to establish equilibrium in the CN-CNO cycle is a significant fraction of the total H-burning lifetime. The convective core starts small, grows to a maximum, and then recedes as usual, independently of the model—either classical or overshoot—used to define the extension of the core. Within a given overshoot scheme, the growth of the core against a gradient in molecular weight is difficult to model. The morphology of the turnoff in the HRD of old clusters (age of a few 10^9 yr) suggests that overshoot cannot exceed a certain extent (Aparicio et al 1990, Alongi et al 1991b, Bertelli et al 1992, Maeder & Meynet 1989). The central H-burning phase of stars lighter than $M_{\rm con}$ is clearly not affected by core overshoot.

In intermediate- and high-mass stars, the overluminosity caused by overshoot during the core H-burning phase still remains during the shell H- and core He-burning phases because of the larger size of the H-exhausted core, $M_{\rm Hc}$. As a consequence of the higher luminosity, the lifetime of the He-burning phase ($t_{\rm He}$) gets shorter in spite of the larger mass of the convective core. This, combined with the longer H-burning lifetime, $t_{\rm H}$, makes the ratio $t_{\rm He}/t_{\rm H}$ fairly low (from 0.12 to 0.06 when the stellar mass varies from $2M_\odot$ to $9M_\odot$). The lifetime ratio is about a factor of 2 to 3 lower than in classical models of the same mass. Since all low mass stars possess nearly identical helium core masses, the inclusion of convective overshoot leads to results similar to those obtained with the classical semiconvective scheme (Bressan et al 1986).

Models of intermediate-mass stars evolved with core overshoot alone produce luminosity functions of main sequence stars that agree much better with the observational data for rich clusters (Chiosi et al 1989a,b); however they hardly match the extension of the blue loops observed in the same clusters (Alongi et al 1991a) because they possess less extended blue loops on the HRD. To overcome this difficulty Alongi et al (1991a) considered the effect of envelope overshoot in addition to that of core overshoot. Envelope overshoot does not alter the scheme determined by core overshoot, but simply makes possible the occurrence of extended blue loops.

Since the evolution of massive stars is heavily dominated by mass loss, the effects of convective overshoot alone are more difficult to single out. These will be examined in greater detail below.

Due to the larger masses of the He and C-O cores left over at the end of core H- and He-burning phases, respectively, the critical masses $M_{\rm up}$ and $M_{\rm HeF}$ are about 30% smaller than in classical models (Barbaro & Pigatto 1984; Bertelli et al 1985; Bertelli et al 1986a,b). The impact of this result on the observational front is straightforward.

Models incorporating core overshoot all along their evolutionary history have not yet been evolved into the TPAGB regime, however we may foresee a behavior qualitatively similar to that of classical models (Chiosi et al 1987). We have already reported that overshoot from the convective shell that follows a thermal pulse has occasionally been adopted to improve upon the explanation of C stars (Hollowell 1988; Hollowell & Iben 1988, 1989, 1990).

5. OLD STAR CLUSTERS

5.1 *Globular Clusters*

The recent revolution in photometric techniques (CCD detectors) dramatically improved the quality of the color-magnitude diagrams (CMD) of globular clusters (GCs) thus allowing comparisons with theoretical models for low-mass stars of unprecedented sophistication. Since an exhaustive referencing to the impressive list of high quality CCD-CMDs is impossible and beyond the scope of this review, we limit ourselves to a few illustrative cases: 47 Tuc (Hesser et al 1987), M3 (Buonanno et al 1987), M62 (McClure et al 1987), and M92 (Stetson & Harris 1988). A very informative review on the data on GCs is by Hesser (1988). There also exists an equally impressive list of theoretical studics for low-mass stars at varying basic parameters: mass, helium abundance Y, metallicity Z (this latter separated into three components [CNO/H], [α]-elements [α/H], and [Fe/H] and their relative proportions), mixing length in the outer convective layer, opacities, nuclear reactions rates, mixing process, diffusion processes, equation of state, neutrino energy losses, mass loss by stellar wind, etc. Most of these models are calculated all the way from the main sequence to the latest stages, thus making available homogeneous sets of evolutionary tracks and isochrones. The most recent reviews on the subject are by Renzini & Fusi-Pecci (1988) and VandenBerg (1991) to whom we refer for an exhaustive referencing. Among the studies presenting extensive grids of stellar models we recall VandenBerg (1983, 1985), VandenBerg & Bell (1985), Sweigart (1987), Caputo et al (1987), Sweigart et al (1987, 1990), Chieffi & Straniero (1989), Straniero & Chieffi (1991), Bencivenni et al (1989), Lee & Demarque (1990), Bergbusch & VandenBerg (1991), Bertelli et al (1990), and Alongi et al (1991b). As thoughtfully discussed by Renzini & Fusi-Pecci (1988), to be safely used in the interpretation of the CMDs the evolutionary models must be tested for accuracy in the input physics and adequacy of the physical assumptions, and finally calibrated using known reference objects (see also VandenBerg 1991 and Fusi-Pecci & Cacciari 1991). Among the various parameters, the calibration of the mixing length in the outer convective layer is particularly important because it affects the luminosity and T_{eff} at the turnoff and position of the RGB. In addition to this, accurate transformations from the theoretical HRD ($M_{\rm bol}$, $T_{\rm eff}$) into an observational CMD (usually in the UBVRI photometry) are required (see Renzini & Fusi-Pecci 1988 and VandenBerg 1991). The adequacy of the physical assumptions (e.g. type of mixing) can be tested by comparing model predictions (e.g. lifetimes) to their observational counterpart (star counts and luminosity functions). It is worth recalling that this comparison is meaningful only if the observations satisfy certain conditions on the minimum number of stars to be sampled. These are well expressed by the *fuel consumption theorem* of Renzini & Buzzoni (1986).

5.1.1 LUMINOSITY FUNCTIONS, MASS FUNCTIONS, AND STAR COUNTS

Luminosity functions (LFs) of the main sequence to very faint magnitudes allows one to test the low-mass main sequence models and to derive information on the present-day mass function (PMF) from which a guess on the initial mass function (IMF) can be obtained. Stars with $M \leq 0.6 M_{\odot}$ are characterized by complex physics which makes it difficult to calculate adequate models. The main reasons are in the adopted model atmospheres, the low temperature molecular opacities, the formation and dissociation of the H_2 molecules, the failure of the ideal gas approximation in the equation of state and adiabatic gradient—specifically the presence of Coulomb interactions (Copeland et al 1970), and the coexistence of partial degeneracy and ionization (Magni & Mazzitelli 1979). The models calculated by VandenBerg et al (1983), D'Antona & Mazzitelli (1986), and D'Antona (1987), although differing in several details of the input physics, predict a sudden flattening of the mass-luminosity relation and steepening of the main sequence below $0.5 M_{\odot}$. Both are due to the effect of the H_2 molecule rather than to the models becoming fully convective. The LF is also expected to steepen at the same luminosity (mass) as perhaps indicated by the observations (see Richer & Fahlman 1991). However, at the present, data are not accurate enough to discriminate among current evolutionary models (see the discussion in Renzini & Fusi-Pecci 1988).

From the earliest studies on the LFs in clusters with different [Fe/H] it was soon evident that the slopes of the LFs were dissimilar, in the sense that metal-rich clusters have LFs with a flatter slope. Interpreted with the aid of theoretical models, because a purely empirical mass-luminosity relation for Population II stars is still lacking, the LFs were transformed into PMFs by means of the usual power-law representation $\phi(m)dm = m^{(1+x)}$, and a correlation between x and [Fe/H] was suggested (McClure et al 1986, 1987). Specifically, the metal-rich clusters should possess a PMF flatter than that of the metal-poor ones. This result has potentially far-reaching consequences, in particular if interpreted in terms of a dependence of the IMF on the metallicity, and therefore it must be examined in detail. First, almost none of the existing LFs extend below $0.5M_{\odot}$; this limit depends however on the adopted stellar models. An exception is given by the LFs presented by Richer & Fahlman (1991), which reach about $0.2M_{\odot}$. Second, the LF obtained in a given portion of the cluster must be corrected to become representative of the whole cluster (see Pryor et al 1986). If internal dynamical evolution and tidal stripping are unimportant, the PMF is simply the IMF because the H-burning lifetime of these stars is much longer than the Hubble time. However, this is not the case because many GCs show clear evidence of internal dynamical interaction and mass segregation (see Pryor et al 1986; also Richer & Fahlman 1991 and references therein) with more massive stars sinking toward the center. Furthermore, tidal stripping may be very efficient (Stiavelli et al 1991). Therefore, as a result of evaporation and stripping, GCs may have lost a large fraction of their low-mass stars with consequent lowering of the slope x. Finally, it has been shown that if observations are taken for very low mass stars at large distances from the cluster core, the observed PMF is similar to the IMF in the absence of extensive tidal stripping (H. B. Richer et al 1991). Therefore the observed PMF in these regions constitutes a lower limit to the IMF. With these limitations born in mind, Ortolani et al (1989), Capaccioli et al (1991), and Piotto (1991) reached the following conclusions: (*a*) the PMF varies from cluster to cluster; (*b*) for intermediate and low metallicity GCs there is no correlation between x and [Fe/H]; (*c*) the most metal-rich clusters have a flatter PMF; and finally, (*d*) the PMF slope seems to correlate with the position of the clusters with respect to the galactic gravitational potential. Specifically, the PMF becomes steeper and steeper at increasing galactocentric distance R_G and height z_G above the galactic plane. The simulations by Stiavelli et al (1991) suggest that tidal disk shocking could be responsible for the observed correlations.

Luminosity functions for the upper main sequence stars (at the turnoff) have been occasionally proposed as age calibrators (Paczynski 1984, Ratcliff 1987, Demarque 1988). However, for the reasons discussed by Renzini (1986, 1988) this method cannot give ages with a precision better than $\pm 5 \times 10^9$ yr.

Star counts of RGB stars in GCs (Hesser et al 1987, Fusi-Pecci et al 1990) have confirmed the existence of a bump in the differential LF, or equivalently a knee in the cumulative LF, whose origin was first pointed out by Iben (1968). As amply discussed by Renzini & Fusi-Pecci (1988), the luminosity of the bump identifies the mass coordinate of the bottom of the homogeneous envelope and, therefore, the maximum penetration of the external convection during the RGB phase. Fusi-Pecci et al (1990), discussing the relationship between the magnitude M_V of RR Lyrae stars and [Fe/H] (see below), noticed that the observed luminosity of the bump is about $0.415 \pm 0.07 M_V$ fainter than predicted by current models. Three causes of disagreement were indicated—old opacities, more efficient envelope convection, and initial abundance of helium—to which different abundances of α-elements, like Ne, Mg, Si, and S can be added (F. Ferraro 1991, private communication). Alongi et al (1991a) argued that opacity and helium abundance cannot rule out the discrepancy, whereas a more efficient mixing at the base of the convective envelope is plausible. Indeed, they found that envelope overshoot of about 0.7 of a pressure scale height H_P can reconcile theory with observations. They also considered the bump luminosity to be an additional constraint to the otherwise uncertain efficiency of this phenomenon. Studies on the effects of the α-elements are underway.

Star counts in RGB, HB, and AGB phases are considered as probes of stellar structure as they reflect the duration of the underlying evolutionary phases hence the adequacy of the physical assumptions in model calculations (Buzzoni et al 1983, Buonanno et al 1985). Four ratios can be constructed: $R = t_{HB}/t_{RGB} \simeq N(HB)/N(RGB)$, $R' = t_{HB}/(t_{RGB}+t_{AGB}) \simeq N(HB)/[N(RGB)+N(AGB)]$, $R_1 = t_{AGB}/t_{RGB} \simeq N(AGB)/N(RGB)$, and $R_2 = t_{AGB}/t_{HB} \simeq N(AGB)/N(HB)$, where all symbols have their usual meaning. Specifically, t_{RGB}, t_{HB}, and t_{AGB} are lifetimes, whereas $N(RGB)$, $N(HB)$, and $N(AGB)$ are star counts. In particular, $N(RGB)$ is the number of RGB stars brighter than the RR Lyrae luminosity, and $N(AGB)$ is the number of AGB stars up to 2.5 magnitude above the RR Lyrae luminosity. The ratios R, R', and R_1 can be used to trace back the helium abundance (Iben 1968) and convective mixing during the HB phase (Castellani et al 1971a,b). The ratio R_2 is particularly sensitive to mixing in HB, and has been found to be able to discriminate among different types of mixing. This ratio is 0.15 ± 0.01 in well studied GCs (Buzzoni et al 1983, Buonanno et al 1985). Contrary to the claim by Renzini & Fusi-Pecci (1988), models with semiconvective mixing cannot match this ratio because several breathing pulses of the convective core prolong the core He-burning lifetime and shorten the AGB lifetime. On the contrary, models with convective overshoot match the above ratio if a plausible efficiency is assumed (Bressan et al 1986, Chiosi 1986).

5.1.2 AGES, AGE SPREAD, AND AGE-METALLICITY RELATION Determining the age of GCs is a complex game (see Demarque et al 1991 and Fusi-Pecci & Cacciari 1991), which requires a knowledge of many parameters, such as the helium content Y, metallicity [M/H], CNO abundance [CNO/H], distance modulus, and reddening.

Since GC stars are too cool to allow direct spectroscopic measurements of the abundance by mass of helium Y, less direct methods are used. A general assumption is that helium abundance in GCs reflects primordial nucleosynthesis as GCs are among the oldest objects in the Universe. In general, the helium abundance is estimated from the big-bang nucleosynthesis (Boesgaard & Steigman 1985, Olive et al 1991), extragalactic HII regions (Davidson & Kinman 1985), the R method (Buzzoni et al 1983), and the HB models themselves (Lee et al 1990). The adopted value is $Y = 0.235 \pm 0.005$ and it is generally assumed to be constant throughout the halo clusters, even if helium abundance has been often considered a candidate for the second parameter (see below).

The metallicity is usually referred to the observed abundance [Fe/H] and the nowadays accepted metallicity scale is from Zinn & West (1984). The majority of GCs have [Fe/H] values from -1.0 to -2.3 dex with typical uncertainty of 0.15 dex. However, recalling that in metal-poor stars, the abundances of Ne, Mg, Si, and S are significantly enhanced with respect to [Fe/H] (Nissen et al 1985), [Fe/H] $\neq$ [M/H], so that [Fe/H] alone is not fully representative of the real content of heavy elements. Useful compilations of [M/H] are from Pilachowski (1984), Webbink (1985), and Hesser & Shawl (1985). The observation of CMDs showing sequences of virtually undetectable width indicates a uniform abundance of heavy elements within the stars of a particular cluster (two exceptions exist: ωCen and M22 which show star to star differences of about 0.5 dex).

It is well known that the morphology of the turnoff greatly depends on the abundance of oxygen. The controversy of the oxygen enhancement in cluster stars as measured by [O/Fe] is still far from being solved. Given [O/Fe] = 0 for the Sun by definition, the questions are whether [O/Fe] is different for the halo stars and whether it varies with [Fe/H]. Observing giant stars in GCs, Pilachowski et al (1983) obtained [O/Fe] = 0.25 regardless of [Fe/H]. A similar estimate is given by Gratton & Ortolani (1989) who find [O/Fe] = 0.40. Therefore, giant stars in GCs seem enhanced in oxygen relative to the Sun with no correlation to [Fe/H]. However, the question arises whether this result holds for all the stars in a cluster, main sequence included, or whether it is limited to giants. An enhancement of the oxygen abundance in giant stars, resulting from inner processes can be excluded on the basis of stellar evolution theory. In dwarf field stars, the ratio [O/Fe] for [Fe/H] ≤ -1 is more controversial. Current estimates for [O/Fe] go from 0.4 to 0.7 dex with no variations with [Fe/H]. The opposite conclusion was reached by Abia & Rebolo (1989) who claimed that [O/Fe] varies from 1.2 for [Fe/H] = -2.3 to 0.6 for [Fe/H] = -1. R. Gratton (1991, private communication) has criticized this result arguing that Abia's & Rebolo's (1989) analysis was very sensitive to the parameters adopted for the atmospheres. The chosen parameters were not appropriate.

A similar conclusion was reached by Spite & Spite (1991). In general, either [O/Fe] = 0 or [O/Fe] $\simeq$ 0.5 to 0.7 is adopted.

Since there are many GCs with low color excess ($E_{B-V} \leq 0.1$) spanning a broad range of metallicities (up to [Fe/H] = -1), reddening is not a serious problem in finding the intrinsic color of the turnoff.

The distance scale of GCs is another topic of strong controversy. Modern determinations of the distance modulus reduce to comparing the apparent magnitudes of the RR Lyrae stars or HB stars with the corresponding absolute visual magnitudes. There are several independent methods to obtain the absolute visual magnitudes of RR Lyrae stars, M_V(RR) (see the discussion by Renzini & Fusi-Pecci 1988 and references therein), which ultimately lead to an assessment of whether or not a correlation between M_V(RR) and [Fe/H] exists and try to fix the slope and zero point of this relation (see also Fusi-Pecci & Cacciari 1991). The method based on the pulsational properties of RR Lyrae stars proposed by Sandage (1982, 1986) gives ΔM_V(RR)/Δ[Fe/H] = 0.35. Specifically, in his investigation of the Oosterhoff effect Sandage (1982, 1986) determined the slope of the above relation from an argument known as the period shift. Assuming that the light curve shapes (rise time) and amplitude of RR Lyrae stars are unique functions of $T_{\rm eff}$, the periods are found to increase with the metallicity according to $\Delta \log P = -0.12\Delta$[Fe/H]. The slope of the M_V(RR)-[Fe/H] relation follows from the pulsation theory assuming that the mass of RR Lyrae is the same irrespective of [Fe/H]. The zero point is derived from an average of Baade-Wesselink and main-sequence fitting magnitudes of RR Lyrae stars: $M_V = 0.89 \pm 0.05$ at [Fe/H] = -1.4 (see Renzini & Fusi-Pecci 1988). Similar slope and zero point, however with a larger range of uncertainty, were recently obtained by Sandage (1990a) by means of a more sophisticated analysis of the problem. [See also the review by Sandage & Cacciari (1990).] Sandage's (1982, 1990a) conclusions seem to be supported by the observational studies of Buonanno et al (1990) and Longmore et al (1990). However, standard calculations of ZAHB models (Sweigart et al 1987) are not able to predict any appreciable period shift, unless helium is anticorrelated with [Fe/H] which likely does not occur [see the discussions by Renzini & Fusi-Pecci (1988), Fusi-Pecci & Cacciari (1991), and Lee (1991a)]. To cast light on the problem, a different analysis was performed by Lee et al (1987), Lee & Demarque (1990), Lee et al (1990), and Lee (1990) who included off-ZAHB evolution and a possible dependence of the $T_{\rm eff}$-pulsation amplitude relation on [Fe/H]. Calculations of synthetic HBs with solar [O/Fe] give ΔM_V(HB)/Δ[Fe/H] = 0.17 and M_V(HB) $\simeq$ 0.70 at [Fe/H] = -1.4. This value for the slope seems to be supported by the observational work of Cacciari et al (1989), Liu & Janes (1990), Fusi-Pecci et al (1990), and Sarajedini & Lederman (1991).

However, this new analysis implies the period shift relation $\Delta \log P = -0.04\Delta[\mathrm{Fe/H}]$, whose slope is much lower than the Sandage (1982) value. Owing to the far-reaching implications of the M_V(RR)-[Fe/H] relation on the age problem (see below), this topic is a matter of debate. For more details on the subject the reader should refer to Renzini & Fusi-Pecci (1988), Sandage & Cacciari (1990), and Lee (1991a).

Rotation does not seem to affect the age in a significant fashion (see Deliyannis et al 1989).

Under the action of gravity, in GC stars helium can sink inward relative to hydrogen. This process may affect the age in two ways. First the lower relative central abundance of hydrogen decreases the main sequence lifetime. Second a higher relative hydrogen abundance in the envelope results in a larger radius (lower T_{eff}) without changing the RGB position. The main sequence turnoff is redder, thus implying that a lower age is required to fit a given cluster (Deliyannis et al 1990, Proffitt et al 1990, Proffitt & VandenBerg 1991, J. Richer et al 1991, Chaboyer et al 1991). The reduction in age is estimated to be about 10–20%.

Given a good CMD, most likely obtained with a CCD detector, ages can be derived by means of the classical isochrone fitting (IF) method, the ΔV method, and the $\Delta(B-V)$ method.

In the IF method, all the parameters discussed above are necessary. Therefore, the ages obtained from isochrone fitting are by far the most uncertain (see also the arguments given by Renzini & Fusi-Pecci 1988 and Fusi-Pecci & Cacciari 1991). Most studies have assumed solar [O/Fe] and find ages going from $10\text{--}12 \times 10^9$ yr for a cluster like Pal 12 ([Fe/H] $= -1.1$; Sarajedini & King 1989, Straniero & Chieffi 1991) to $16\text{--}18 \times 10^9$ yr for clusters like M92 and M68 ([Fe/H] $= -2.1$; King et al 1988, Stetson et al 1989, Straniero & Chieffi 1991). If [O/Fe] varies with [Fe/H], this age range is less clear. If helium diffusion is included, an age reduction of 2×10^9 yr is possible, as estimated by Chaboyer et al (1991) for the cluster NGC 288.

The ΔV method rests on the fact that the turnoff magnitude becomes fainter as a cluster evolves, while the HB luminosity is virtually constant. ΔV is the magnitude difference between the turnoff and the HB at the turnoff color. This method is independent of reddening. Furthermore, the magnitude of the RR Lyrae stars and turnoffs are likely scarcely dependent on [O/Fe] and helium diffusion. The disadvantage with this method is that not all GCs possess RR Lyrae stars, and some HBs are not horizontal. Furthermore, the turnoff is almost vertical, which makes uncertain the definition of the turnoff magnitude as well. It requires an assumption for the helium abundance. Finally, there is the effect of the controversial relations M_V(RR)-[Fe/H] and [CNO]-[Fe/H] (see above). On the observational side, ΔV does not correlate with [Fe/H] (Buonanno et al 1989 and references therein), but there is some scatter in ΔV at given metallicity. According to Buonanno et al (1989) $\Delta V \simeq 3.54$. An overview of the possible alternatives one may have from the different combinations of the slopes and zero points of the above relations is given by Fusi-Pecci & Cacciari (1991) to whom we refer in the summary below. With [CNO/Fe] = 0 and $\Delta M_V(\mathrm{RR})/\Delta[\mathrm{Fe/H}] = 0.35$, all clusters are coeval and no significant age-metallicity relation exists. With [CNO/Fe] = 0 but $\Delta M_V(\mathrm{RR})/\Delta[\mathrm{Fe/H}] = 0.20$, the cluster ages decrease with increasing metallicity by about 4×10^9 yr (Sarajedini & Demarque 1990, Sarajedini & King 1989, Lee et al 1990, Sandage & Cacciari 1990). The same is true if $M_V(\mathrm{RR})$ is independent of [Fe/H] and equal to the classical value of 0.6 (Sarajedini & King 1989). Because of the period shift effect, this alternative is less probable. The zero point of the M_V(RR)-[Fe/H] is however crucial to setting the scale of the absolute ages, but unfortunately it is uncertain. With [CNO/Fe] ≥ 0 the situation is more complicated. If [CNO/Fe] ≥ 0 but independent of [Fe/H], $\Delta M_V(\mathrm{RR})/\Delta[\mathrm{Fe/H}] = 0.35$, and $\Delta V = 3.54$, then all clusters are coeval but the absolute ages decrease with respect to the case with [CNO/Fe] = 0 by a quantity depending on the degree of CNO-enhancement (e.g. with [CNO/Fe] = 0.3 the ages are decreased by about 1×10^9 yr). Keeping constant all other relations but letting [CNO/Fe] increase with decreasing [Fe/H], the condition $\Delta V = 3.54$ does not imply coevality of GCs. Due to the differential enhancement of [CNO], all metal-poor clusters are younger, while the age of the metal-rich ones are only marginally decreased. However, this combination of slope and abundances may lead to the following two indications against intuition: Y anti-correlates with [Fe/H] and the metal-poor clusters are younger than the metal-rich ones. If $\Delta M_V(\mathrm{RR})/\Delta[\mathrm{Fe/H}] = 0.20$ and $\Delta V = 3.54$ the following cases are possible. With [CNO/Fe] ≥ 0 but constant at varying [Fe/H] all clusters have ages decreasing with increasing [Fe/H]—the metal-rich ones are the youngest. The absolute ages depend on the degree of [CNO]-enhancement as above. If [CNO] increases with decreasing [Fe/H], all the clusters may be coeval for a suitable difference in the [CNO]-enhancement between metal-rich and metal-poor clusters. Finally, the inclusion of helium diffusion in model calculations would lead to even lower ages without changing the above scheme.

The $\Delta(B-V)$ method is based on the color difference between the turnoff and the base of the RGB (Sarajedini & Demarque 1990, VandenBerg et al 1990). This color difference decreases as the cluster age increases. Assuming that the mixing length used in stellar models is calibrated, the method is independent of distance, reddening, photometric zero point, helium abundance, and, to first order it seems to be insensitive to variations in [Fe/H]. The major uncertainties are with the transformations from $T_{\rm eff}$ to colors, the degree of helium diffusion, and [O/Fe], all of these affecting the turnoff color. $\Delta(B-V)$ is reduced by an increased age, an enhancement in oxygen abundance, and helium diffusion. According to VandenBerg et al (1990) this method is particularly suited to determine relative ages as the determination of absolute ages is affected by the uncertainties in the convection theory and color transformations. Using the revised Yale isochrones (Green et al 1987) and assuming [O/Fe] = 0, Sarajedini & Demarque (1990) and Sarajedini (1991) find that the age of the oldest clusters is about 18×10^9 yr and indicate that GCs span an age range of at least 2.5×10^9 yr. Enhancement of [O/Fe] and/or helium diffusion would reduce the age by about 2×10^9 yr (see Proffitt & VandenBerg 1991). As far as the age spread among clusters with similar metallicity is concerned VandenBerg et al (1990) give the following indication. The most metal-poor clusters ([M/H] = -2.1) are uniform in age within 0.5×10^9 yr; clusters with [M/H] = -1.6 are also coeval though some age spread cannot be excluded; finally the most metal-rich clusters, [M/H] ≥ -1.3, appear to encompass a significant range. This indicates that the age spread increases with the metallicity as expected if the collapse of the halo was of prolonged rather than of short duration ($\leq 1 \times 10^9$ yr).

Absolute ages, age spread, and age-metallicity relation of GCs are significant to cosmology and galaxy formation. The oldest GCs set a lower limit to the age of the Universe, whereas the age spread and age-metallicity relation, if real, not only could be a solution to the problem of the second parameter controlling the morphology of the CMDs of GCs, but also constrain the time scale and mechanism of halo formation. Long ago Searle & Zinn (1978) made the hypothesis that age is the second parameter driving the morphology of HBs (the metallicity is the first). Other second parameter candidates, such as Y, [CNO/Fe], or core rotation have been considered that could also account for the observed differences (see Renzini 1977; Lee 1991a,b) but to date only the age seems to provide an explanation compatible with both the standard theory of stellar evolution and the observed distribution of RR Lyrae stars. Two ideal clusters for testing the possibility that the age is the second parameter are NGC 288 and NGC 362. These clusters have similar [Fe/H] (-1.4 and -1.28 respectively; Zinn 1985) but totally different HBs. Another pair is provided by NGC 6397 (see Demarque et al 1991) and Ruprecht 106 (Buonanno et al 1990) with [Fe/H] = -1.9. Analyzing these pairs, Sarajedini & Demarque (1990) and Demarque et al (1991) find the age difference of about 3×10^9 yr for the first pair and 4×10^9 yr for the second one. They also argue that the age is the second parameter of GCs. A similar conclusion was reached by Bolte (1989). VandenBerg & Stetson (1991) using the $\Delta(B-V)$ method argue that the pair NGC 362–NGC 288 shows a difference in $\Delta(B-V)$ consistent either with an age difference of 2×10^9 yr or with a difference in [O/Fe] of 0.6, and consider premature the identification of the age as the second parameter.

Because an important characteristic of the second parameter phenomenon is its systematic variation with the galactocentric distance (Searle & Zinn 1978), Lee (1991b) sought for a global interpretation of the available information correlating [Fe/H], [CNO/Fe], HB type, galactocentric distance, and relative ages of GCs. In the Lee (1991b and references therein) scenario, very likely the age is the second parameter that has the largest influence in determining the HB morphology, and the clusters in the inner halo ($R_G \leq 8$ Kpc) are in the mean several billion years older than the outer halo clusters. At the same time, arguments are given that run counter to the hypothesis that helium abundance, core rotation, or [CNO/Fe] abundance are the second parameter. If this interpretation is correct, it lends support to the idea of prolonged phase of Halo formation, possibly involving mergers and accretion of large fragments with independent dynamical and nucleosynthetic histories (Larson 1990). It is worth recalling that, as recently pointed out by Sandage (1990b), a significant age spread among GCs does not contradict the picture of Galaxy formation suggested long ago by Eggen et al (1962).

Although absolute ages are less important from the point of view of interpreting the CMD of GCs, they are a key constraint to the minimum age of the Universe. The above discussion has clarified that the absolute age depends very strongly on the accuracy and adequacy of both observational parameters and stellar models (see the discussion by VandenBerg 1991). Therefore, the absolute ages are subject to change as soon as one of the basic parameters is improved. The ages estimated by Sarajedini & King (1989) for a selected sample of GCs show that their distribution peaks at about $16(\pm 2) \times 10^9$ yr, with wings going down to 10×10^9 yr and up to 20×10^9 yr.

5.2 *Open Clusters*

The old open clusters, whose ages range from say 1 to 7–8 $\times 10^9$ yr, trace most of the history of the Galactic Disk. Therefore, the correct ranking of old open clusters as a function of age, chemical composition, and kinematical properties, is of paramount importance to understanding the process of star formation in the Galactic Disk. Furthermore, having turnoff masses between $1 M_\odot$ and $2 M_\odot$, they are probes of stellar structure in that mass range, in which the transition from radiative to convective cores on the main sequence, from pp chain to CNO cycle for the core H-burning phase, and from very bright RGBs as in M67 to much less evident RGBs as in the Hyades occur. We will limit ourselves to discuss problems related to the structure of these stars. As first pointed out by Barbaro & Pigatto (1984), the interpretation of the CMD of these clusters (e.g. NGC 2420, NGC 3680, IC 4651, King 2, King 11, M67, etc) in terms of the classical models encountered some difficulties that could be solved by invoking a certain amount of convective overshoot during the main sequcnce core H-burning phase and hence older ages with respect to those from classical models. The main signatures are the detailed shape of the main sequence turnoff, the shape of the RGB, the clump of red stars (most likely core He-burners), and the number of stars brighter than the main sequence at the beginning of the subgiant branch with respect to the main sequence stars (see Mazzei & Pigatto 1988; Maeder 1990; Antony-Twarog et al 1988, 1989, 1990; Andersen & Nordstrom 1991; Aparicio et al 1990). Another type of evidence comes from eclipsing binaries, for which good determinations of mass, radius, luminosity, and abundances are available (Andersen et al 1990), falling near the turnoff of some of these clusters. Specifically, binary stars with small convective cores ($M = 1.2M_{\odot}$) are very well fitted by standard models, while at slightly larger masses $1.5M_{\odot} \leq M \leq 2.5M_{\odot}$, the moderately evolved binaries require a certain amount of convective overshoot. This agrees with the scheme proposed by Aparicio et al (1990), Maeder & Meynet (1991), and Alongi et al (1991b), in which the efficiency of convective overshoot during the core H-burning phase of stars in this mass range is suggested to gradually increase with the star mass. A similar study was made by Napiwotzki et al (1991) for the somewhat younger cluster NGC 2301 (estimated age of a few 10^8 yr) for which a careful determination of $T_{\rm eff}$s and gravities for the brightest members of the cluster were available. Since four out of five stars fall beyond the limit for the core H-burning phase of classical models, a cooler turnoff of the main sequence seems to be required. This was attributed to substantial overshoot. The result was criticized by Brocato & Castellani (1991) who claimed that their recent models with the classical scheme (Castellani et al 1990, 1991) possess the required extension in the CMD. However, the main sequence extension in the Brocato & Castellani (1991) HRD is not too different from the classical one shown by Napiwotzki et al (1991). This implies once again that with classical models too many stars are in the short-lived phase of shell H-burning. Further support to this scheme comes from the careful analysis of the CMD of IC 4651 by Bertelli et al (1992) who adopted both classical and overshoot models using the same input physics (Fagotto 1990, Alongi et al 1991b).

A recent determination of ages for a selected sample of old Galactic clusters, including the Sandage (1988) list, is by Carraro (1991). This study collects the most recent CMD of each cluster, adopts the compilation of metallicities by Friel & Janes (1991), makes use of both classical and overshoot models calculated by the Padova group (Fagotto 1990; Alongi et al 1991a,b), and finally relies on the synthetic CMD technique (see Chiosi et al 1989b) instead of the simple isochrone fitting to estimate reddening, distance modulus, and age at the same time. The cluster ages span from 0.9×10^9 yr for NGC 2477 to 8×10^9 yr for NGC 6791.

6. INTERMEDIATE AGE CLUSTERS AND CEPHEID STARS

6.1 *Star Clusters as Templates of Stellar Models*

The young rich clusters of the Large Magellanic Cloud (LMC) are classical templates to which the results of stellar evolution theory for intermediate-mass stars are compared. Because of the large number of stars contained in these clusters, it is possible to make meaningful comparisons even for the shortest lived evolutionary phases. A powerful workbench is NGC 1866, a type III cluster in the classification of Searle et al (1980), whose total mass is estimated in the range $3.6–5 \times 10^5\ M_\odot$. This cluster is well populated throughout the various evolutionary phases, exhibits an extended loop of giant stars, and is rich in Cepheids (Walkcr 1987, Welch et al 1991). First attempts to interpret the CMD of NGC 1866 date from Arp (1967), Hofmeister (1969), and Robertson (1974). Becker & Mathews (1983) using the Robertson (1974) CMD noticed two important features. First, for the observed luminosity of the giants, there are too many stars above the predicted main sequence turnoff—a significant fraction of the number of giant stars. Second, the predicted ratio of post main sequence stars to the main sequence stars was about four times the observed one. Bertelli et al (1985) using the same CMD concluded that only models with convective overshoot could overcome the difficulty. More recent CCD data (Chiosi et al 1989a, Brocato et al 1990) and new stellar models reopened the question whether models with overshoot ought to be preferred to the classical ones or to those with semiconvection. Chiosi et al (1989b), using both the standard models by Becker (1981), which do not incorporate any special treatment of central convection, and models with overshoot by Bertelli et al (1985, 1986a,b), gave the following results: The turnoff mass and age were $5M_\odot$ and 70×10^6 yr with the former models, and $4M_\odot$ and 200×10^6 yr with the latter, respectively. Because the evolutionary tracks alone could not show unambiguously which of the evolutionary schemes was correct, Chiosi et al (1989b) made use of the integrated luminosity function of the main sequence stars normalized to the number of giants (NILF) as a way to achieve the goal. This is possible because the

NILF simply reflects the ratio of core He- to H-burning lifetimes, which depend on the stellar models in use. The conclusion was that models with substantial core overshoot reproduced the observed NILF, whereas classical models failed. Similar analysis, repeated using models with semiconvection (Lattanzio et al 1991), reached identical conclusions. The analysis was extended to other clusters of the LMC, like NGC 1831 (Vallenari et al 1991a) and NGC 2164 (Vallenari et al 1991b) with similar results, i.e. models with core overshoot provided a good fit to the CMD and luminosity functions at the same time. Brocato et al (1990) obtained CCD photometry of NGC 1866 and analyzed the CMD and LF following the method outlined by Chiosi et al (1989b) but using the models without convective overshoot calculated by Castellani et al (1990, 1991). They came to the conclusion that core overshoot is not required, if one makes use of modern opacities (see Castellani et al 1990, 1991). Although the opacity may lower the ratio of core H- to He-burning lifetimes from 0.33 (Becker 1981) to 0.23, this value is still far from that indicated by the observations (0.10) or given by models with overshoot. Bressan (1990) clearly showed that Brocato's et al (1990) conclusion was entirely due to the different luminosity function, a point of embarrassment because similar observing and reducing techniques were used. The reason for the disagreement is likely to be the different number of red giants in the two samples. Bencivenni et al (1991), analyzing the much younger cluster NGC 2004 (the age and turnoff mass of which are a few 10^6 yr and about $20 M_\odot$, respectively), claim that arguments can be given against the existence of convective overshoot. However, we recall that in this mass range the inclusion of convective overshoot does not bring a significant difference with respect to the classical models (e.g. the core He- to H-burning lifetime ratio is modestly changed). Furthermore, the evolution is dominated by mass loss, which was not taken into account by Bencivenni et al (1991). Many observational tests of convective overshoot in intermediate-mass stars have been critically scrutinized by Stothers (1991). Defining the ratio $d/H_{\rm p}$ of the effective convective overshoot distance beyond the classical Schwarzschild boundary to the local pressure scale height $H_{\rm p}$ as an index characterizing published models, Stothers (1991) comes to the conclusion that $d/H_{\rm p} \leq 0.4$ is likely an upper limit to this phenomenon. This estimate is comparable to the value adopted by Maeder & Meynet (1991 and references therein), and Alongi et al (1991a,b) in their recent model calculations. Despite the net improvement of observational CMDs and LFs, the situation appears to be rather confused simply reflecting the difficulty of producing accurate and adequate stellar models with correct lifetimes for the core H- and He-burning phases.

6.2 *Pulsational Models of Cepheid Stars*

In recent years, there have been a considerable number of photometric studies of Cepheids in the field of the LMC and SMC (e.g. Caldwell & Coulson 1986 and references therein) and there is currently much observational effort being put into the search for, and study of, Cepheids in the rich star clusters of the Magellanic Clouds (e.g. Mateo et al 1990, 1991; Welch et al 1991). Because they lie at the same distance, the Cepheids in the Magellanic Cloud clusters are basic to two important topics of astronomy: the understanding of pulsation theory itself and stellar evolution theories in general, and the testing of current calibrations of the cosmic distance scale through the calibration of the period-luminosity-color (PLC) relation (Sandage 1958, Sandage & Tammann 1968, Schmidt 1984, Feast & Walker 1987, Walker 1988, van den Bergh 1989, Madore & Freedman 1991). Furthermore, modern observations are done increasingly towards the red using BVRcIc photometry rather than the more traditional BV photometry (see Madore & Freedman 1991). Most of the theoretical modeling of the Cepheid pulsation rests on the pioneer work of Iben & Tuggle (1972a,b; 1975) and Becker et al (1977). [See the reviews by Becker (1985) and Chiosi (1989, 1990).] However, these calculations covered a limited range of masses, were based on old models for intermediate-mass stars, and were made mostly at solar metallicity ($Z = 0.02$). Recent models of Cepheid stars were calculated by Chiosi & Wood (1991) in the mass range $3M_\odot$ to $12M_\odot$ and with chemical abundances appropriate for the solar vicinity, LMC, and SMC, i.e. $Y = 0.25$ and $Y = 0.30$, and $Z = 0.016$, $Z = 0.008$, and $Z = 0.004$ (e.g. Russell & Bessell 1989). In addition to this, Chiosi & Wood (1991) analyzed the response of pulsation to different schemes for the evolution of intermediate-mass stars, i.e. for classical models, models with mild core overshoot, and models with full core overshoot. For each model, three modes of pulsation were calculated: fundamental, first overtone and second overtone. They adopted the radiative opacities by Huebner et al (1977) plus the molecular contribution by Alexander (1975), and Alexander et al (1983) according to the prescription by Bessell et al (1989) and the revision by P. R. Wood (1990, unpublished). Finally, the luminosities and $T_{\rm eff}$s of the models were converted to magnitudes and colors in the BVRI passbands with the aid of either the Green et al (1987) scale or tables amalgamating data from Bell & Gustafsson (1978), Gustafsson & Bell (1979), Buser & Kurucz (1978), and R. Buser (1989, unpublished). For more details see Chiosi & Wood (1991) and Chiosi (1991).

The blue edges of the instability strips of Chiosi & Wood (1991) agree with the corresponding ones of Iben & Tuggle (1972a,b; 1975), whereas the red edges have a different inclination whose slope varies with the metallicity. Red edges not running parallel to the blue ones have been suggested by Fernie (1990) for the Galactic Cepheids and are perhaps confirmed by the observational study of Mateo et al (1991 and references) of Cepheids in LMC clusters. Chiosi et al (1992a), comparing the Fernie (1990) empirical instability strip to theoretical predictions obtained from the Chiosi & Wood (1991) Cepheid models and the synthetic CMD technique, showed that both the edges and the distribution of stars within the strip could be reproduced.

Chiosi & Wood (1991) also presented the period-luminosity (PL) and PLC relationships in the BVRI passbands for the three harmonics and the various compositions. The PL relations agree well with the observational ones (see Feast & Walker 1987) and their zero points are nearly independent of the chemical composition as indicated by the analysis of observational data by Madore & Freedman (1991). The PLC relations have the period term in good agreement with the observational determinations (see Feast & Walker 1987, Madore & Freedman 1991), whereas the color term is larger than estimated by Caldwell & Coulson (1986) and closer to the early estimate by Sandage (1958). The reason for the difference is not understood.

6.3 *Mass Discrepancy of the Cepheid Stars*

It has long been debated whether the masses determined from stellar evolution theory agree with those derived from pulsation theory (see Iben 1974; Iben & Tuggle 1972a,b, 1975; Cox 1980, 1985). In general, pulsational masses ($M_{\rm pul}$) are estimated to be 30 to 40% lower than evolutionary masses ($M_{\rm evol}$) of the same luminosity. The mass discrepancy problem can be reduced to the following causes, each of which is affecting the masses in question in a different way (see the reviews by Becker 1985; Cox 1980, 1985; Pel 1985; and Iben & Tuggle 1972a,b, 1975): (*a*) significant mass loss at some point between the main sequence and the Cepheid stage could decrease $M_{\rm evol}$; (*b*) uncertainties in the determination of the distance of the Cepheid stars would affect largely $M_{\rm pul}$ and to a lesser extent $M_{\rm evol}$; (*c*) uncertainties in the conversion from colors to $T_{\rm eff}$s would affect both $M_{\rm pul}$ and $M_{\rm evol}$; (*d*) inadequacy of the pulsation theory which would obviously reflect on $M_{\rm pul}$; finally, (*e*) inadequacies of current stellar models which bear on the determination of $M_{\rm evol}$.

The effect of mass loss was studied by Willson (1988) and Willson & Bowen (1984), who suggested that stellar winds, somehow triggered by the pulsational instability itself, should occur while the star is within the instability strip. Evolutionary calculations by Brunish & Willson (1987), including mass loss during the Cepheid stage, confirm that Cepheids are trapped in the instability strip, mass loss can continue over a relatively long time scale, and the total mass is significantly reduced, whereas the luminosity is about constant. The rates of mass loss required by these model calculations are of the order of $7\times10^{-9}M_\odot/\mathrm{yr}$ for a $5M_\odot$ star and $2\times10^{-7}M_\odot/\mathrm{yr}$ for a $7M_\odot$ star. Unfortunately, efforts to observe Cepheid winds directly have given so far inconclusive results (see Willson 1988). Finally, the amount of mass loss during the Hayashi line is negligible according to the current mass-loss rates (de Jager et al 1988).

As far as the calibration of the distance scale and the conversion from colors to T_{eff}s are concerned the reader is referred to Pel (1985) for further details.

We have already recalled that period ratios at evolutionary masses and luminosities can be explained by an enhancement in the opacity (Simon 1982, 1987), like that found in modern opacity calculations. Whether this can rule out the mass discrepancy in the classical sense has not yet been investigated.

As already discussed above, convective overshoot alters the mass-luminosity relationship of core He-burning models. Thus, at any given initial mass, the tracks cross the instability strip at higher luminosity than classical models, or conversely, at any given luminosity the corresponding Cepheid mass is significantly lower (Matraka et al 1982, Bertelli et al 1985). Once again, the star clusters of the LMC with Cepheids are the ideal workbench, because all the stars lie at the same distance and membership is less of a problem. This topic has been examined in great detail by Chiosi et al (1992b) using the Cepheid stars and CMD and NGC 2157 (Mateo et al 1990). On the one hand, the fit of the CMD with theoretical simulations based either on classical models or models incorporating core overshoot leads to accurate determination of the M_{evol} of the Cepheid stars, together with the age and chemical compositions. On the other hand, the use of a well calibrated relation between mass-period-luminosity-color (MPLC) for Cepheid stars with the chemical composition suited to the cluster in question, allows a good determination of M_{pul}, M_{evol}, and distance modulus to the LMC at the same time. This analysis indicates that the problem of mass discrepancy likely originates from the adoption of classical models, i.e. without overshoot, to derive M_{evol}, and from the lack of sufficient accuracy in the determination of the distance which bears on both M_{pul} and M_{evol}. The resulting distance modulus to thc LMC is $(m-M)_o = 18.5\pm0.1$ in agreement with the recent determination by Panagia et al (1991) based on the circumstellar ring observed by the Hubble Space Telescope around supernova 1987A in the LMC.

7. SUPERGIANT STARS IN THE MILKY WAY AND LMC

The most recent HRDs for the supergiant stars in the solar vicinity and LMC have been published by Blaha & Humphreys (1989) and Fitzpatrick & Garmany (1990), respectively. Although these samples are based on different selection criteria (spectral types for the Galactic supergiants and two-color photometry for the LMC supergiants), and suffer from a certain degree of incompleteness difficult to assess, in particular among the earliest and latest spectral types, they have several common features, described below.

The luminosity of the brightest blue stars is about a factor of six higher than that of the luminous red stars [a well known result first pointed out by Humphreys & Davidson (1979)]. This defines a luminosity boundary, whose implications are obvious. Since stars evolve from the main sequence towards cooler $T_{\rm eff}$s at roughly constant luminosity (Chiosi et al 1978, Chiosi & Maeder 1986), the observed absence of stars to the right of the boundary may mean that either stellar evolution proceeds so fast that the probability of observing stars with $M \geq 50 M_{\odot}$ is very low, or that the most luminous blue stars never become red supergiants but evolve directly into WR stars. Humphreys & Davidson (1979) suggested that the boundary is due to an instability encountered by the most massive stars as they evolve away from the main sequence and it is somehow related to mass loss. As a matter of fact, the highest mass-loss rates are observed along the boundary (de Jager et al 1988). The boundary itself is marked by the presence of some very luminous unstable stars, known as the LBV. The physical cause of the boundary is usually assumed to be set by the balance between the acceleration due to gravity and the Eddington gradient of radiation pressure. However, to explain the temperature dependence of the boundary for the hot stars, other effects must be included. The work of several investigators (Humphreys & Davidson 1984; Appenzeller 1986; Lamers 1981, 1986; Lamers & Fitzpatrick 1988; Davidson 1987; de Jager 1984; Boer et al 1988; Piters et al 1988; de Koter et al 1988; Carpay et al 1989) has shown that the boundary (stability limit) is the consequence of radiation pressure (modified Eddington limit accounting for variations in the opacity) for the hot stars, and a turbulent pressure gradient in the atmospheres for the cool stars.

At somewhat lower luminosities the density of stars in the HRD shows a distinct decrease redward of $3.9 \leq \log T_{\rm eff} \leq 4.2$ and the density dropoff forms a diagonal line, otherwise called the "ledge" (Fitzpatrick & Garmany 1990), going to lower luminosities at decreasing $T_{\rm eff}$. The most plausible explanation of the ledge is that stars of initial mass up to about $40\text{–}50 M_{\odot}$

either (*a*) perform an extended blue loop in the HRD before core He-exhaustion, as in models with semiconvection evolved at constant mass (Chiosi & Summa 1970) and in models with semiconvection and mass loss by stellar wind (Langer et al 1989, Langer 1991), or (*b*) behave as the models by Brunish & Truran (1982a,b) that slowly move redward in the HRD diagram during the whole core He-burning phase and at the end of this quickly move to the red supergiant region. These models are at the base of the explanation of the blue ledge advanced by Tuchman & Wheeler (1990), who favor models without core overshoot, whereas Fitzpatrick & Garmany (1990) argue that the basic observational restrictions seem to suggest that the true mode of evolution probably resembles, at least qualitatively, either case A or case B scenarios proposed long ago by Chiosi & Summa (1970). On the contrary, current models with mass loss and core overshoot alone (Maeder & Meynet 1987, 1988, 1989, 1991; Maeder 1990) cannot solve the problem because their whole core He-burning phase occurs in the red supergiant region. Therefore, not only the ledge but also the existence of stars in the middle of the HRD cannot be explained. However, the inclusion of overshoot at the base of the convective envelope (Alongi et al 1991a) favors extension of the loops and new models for massive stars (Alongi et al 1991b) well reproduce the shape of the blue ledge. Moreover these models are able to account for the observations of CNO processed and He-rich material at the surface of some He-rich objects near the main sequence (Kudritzki et al 1983, 1989; Bohannan et al 1986).

The population of red supergiants is distinctly separated from all remaining stars in the HRD by the so-called Red Hertzsprung Gap (RHG) showing that the stars must cross the region between the late G supergiants and the M supergiants on a very short time scale. The maximum luminosity attained by the red supergiant stars is about $M_{\rm bol} = -9.5$. There are a few differences between the population of Galactic and LMC M-type supergiants that can be ascribed to the different chemical compositions.

The ratio of red to blue supergiants in the luminosity range $-9.5 \leq M_{\rm bol} \leq -6.5$ is about 1:10 (Humphreys & McElroy 1984). Since the generally accepted idea is that the vast majority of red supergiants are genetically linked to the massive blue stars, rather than AGB stars coming from lower ranges of mass (Brunish et al 1986), the obvious implication is that in the above luminosity range the evolutionary models must account not only for the existence of a rich population of blue stars but also for the red supergiants. How this can be achieved by stellar models is not very clear. The most popular view is that red supergiants are in some early stages of core He-burning although the alternative of late stages of the same phase cannot be excluded.

The star counts in different areas of the HRD of supergiant stars by

Bressan et al (1981), Meylan & Maeder (1982), Bertelli et al (1984), Vanbeveren (1987), and Tuchman & Wheeler (1990) indicate that in spite of the success of the current models with mass loss and core overshoot (see Chiosi & Maeder 1986) in understanding the connection between O, Of, B through M, WR stars, and luminous blue variables (LBV) many properties of massive stars are still far from being fully clarified. Specifically, there is an evident lack of massive stars ($M \geq 40 M_{\odot}$) near the theoretical zero-age main sequence. Such an effect has been also noticed by Garmany et al (1982) using a different sample of stars. The comparison with theoretical isochrones indicates that the youngest known H-burning O-type stars have an age of about $1\text{–}2 \times 10^6$ yr so that about 20% of the core H-burning lifetime is not observed. This agrees with the empirical estimate by Wood & Churchwell (1989) that about 10–20% of all O stars in the solar vicinity are embedded in molecular clouds, and therefore only indirectly detectable by their interaction with the surrounding medium. Only after about 10^6 yr from the start of central H-burning, does the circumstellar gas and dust become transparent and the star become visible. Since the 10–20% fraction is comparable to or even greater than the total fraction of stars in post main sequence stages, this point, together with that of the photometric completeness of the data sets, are crucial in the comparison of theoretical models to the observed star frequencies. In addition to this, while 10–20% of the theoretical lifetime of a star is spent outside the main sequence band, the star counts indicate that some 40% of the stars fall outside this region. Moreover they populate a region just to the red of the main sequence band, the so-called Blue Hertzsprung Gap (BHG), which should be depopulated according to the theoretical models. In order to reconcile the theory with observations, one may suppose that either the data sets are severely biased by incompleteness and/or selection effects or that the main sequence band has to extend to at least the spectral type B9. Among the possible causes of main sequence widening and/or changes in the core H- to He-burning lifetime ratio, Bertelli et al (1984) and Nasi & Forieri (1990) investigated the effects of mass loss by stellar wind, atmospheric effects on the stellar radius caused by mass loss, convective overshoot from the inner core, and a suitable increase in the standard radiative opacity in the region of the CNO ionization. Such an increase in the opacity is today confirmed by the opacity calculations of Iglesias et al (1990) and Iglesias & Rogers (1991a,b). However, the preliminary model calculations by Bressan et al (1991) with the new opacities show that their effect on models of massive stars is small. The above problem still remains. Finally, Tuchman & Wheeler (1990) examined the loci of stationary core He-burning in the HRD (see also Tuchman & Wheeler 1989) and conclude that the distribution of stars cooler than $\log T_{\rm eff} = 4.3$, i.e. beyond the

BHG, is consistent with the theoretical expectation from models without core overshoot, like those of Brunish & Truran (1982a,b), which are known to ignite helium in the core at high $T_{\rm eff}$ and evolve at gradually increasing speed toward the red where only the lifetime (short indeed) of the post He-burning phases is available to account for the red supergiants.

The nature of the stars falling inside the BHG is still a matter of debate. They could result also from inadequacies of the conversion from spectral type, color, and apparent magnitudes into $T_{\rm eff}$s and luminosities, and from uncertainties in the estimate of the individual reddening and color excess. This is particularly important for the LMC stars, whose $T_{\rm eff}$s and luminosities are almost entirely based on two-color photometry, whereas it is less critical for the Galactic stars for which spectral types are available. It follows from this that large uncertainties affect the $T_{\rm eff}$ and hence the positions of the stars in the HRD, in particular in the BHG, whose width in the $B-V$ color is comparable to the uncertainty in the color itself (Chiosi et al 1991). One may argue that inhomogeneities in the chemical composition of Galactic as well as LMC supergiants or the use of enhanced opacities in the models (Iglesias et al 1990, Iglesias & Rogers 1991a,b), by affecting the location of the red edge of the main sequence band and of the blue edge of the core He-burning band, could result into an apparent filling up of the Hertzsprung gap. It can be easily seen with aid of published evolutionary models of different composition (Maeder 1990) and models in which the effects of the new opacities have been tested (Bressan et al 1991) that the gap, although less extended in $T_{\rm eff}$, cannot be eliminated. Finally, Tuchman & Wheeler (1990) advance the hypothesis that a large fraction of these stars are secondaries that have accreted He-rich matter from the envelope of a red supergiant primary, and suggest that about 90% of the stars in the gap should be He-rich. They argue that the abundance determinations by Kudritzki et al (1989) of a few stars in this region found to be He-rich ($Y \geq 0.5$) support this idea.

Additional constraints to understanding the evolution of massive stars are posed by the occurrence of SN1987A. The basic requirement is that SN1987A had a blue progenitor ($\log T_{\rm eff} = 4.0$) of initial mass of about $20 M_{\odot}$ (Arnett et al 1989), which underwent significant surface enrichment of He and C/N elements shortly before the explosion (Fransson et al 1989). The observations indicate that it followed a blue-red-blue evolution and that the last excursion to the blue took place shortly after the central He-exhaustion (Fransson et al 1989). Current models with core overshoot, mass loss, and chemical abundances in the range appropriate for the LMC (Maeder 1990) can start the C-burning phase in the blue only with an envelope mass much less than that deduced for the SN1987A progenitor. As far as models with semiconvection is concerned, the use of the Ledoux criterion (Woosley 1988, Weiss 1989b) in constant mass models leads to a blue progenitor. However, when mass loss is included, these models evolve to high $T_{\rm eff}$s during the mid core He-burning phase, in contrast with the observational suggestion that the SN1987A precursor was a red supergiant a few thousand years before the explosion. Models with a semiconvective treatment intermediate between the Ledoux and the Schwarzschild criterion (Langer et al 1989, Langer 1991) account for the blue progenitor, whereas mixing induced by rotation and mass loss by stellar wind in the previous phases secure He and C/O enrichment at the surface. On the one hand the new semiconvective models may lead to a solution of the SN1987A puzzle; on the other hand the lifetimes of the major nuclear phases (core H- and He-burnings) are not significantly different from those of the old ones, and they would run immediately into the same difficulties encountered by these latter in interpretation of the global properties of the HRD of supergiant stars (main sequence width, star counts, etc). Unless SN1987A is an exceptional event, a solution must be found keeping in mind that the same evolutionary models must account for the properties of this supernova as well as the population of supergiant stars.

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