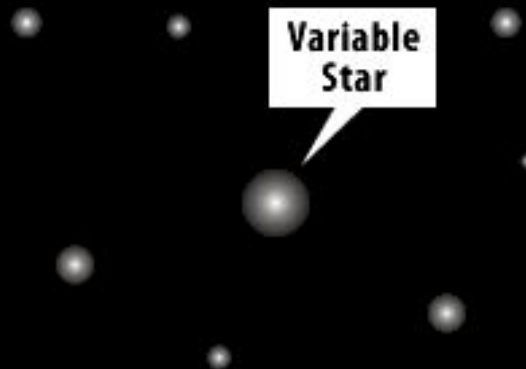


# Variabilities & Instabilities

## ★ in Stars ★

Priv.-Doz. Dr. Kerstin Weis  
Astronomisches Institut RUB

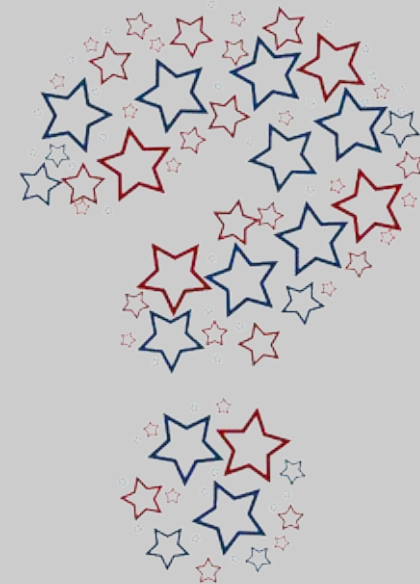


**star formation**

**stellare parameters**

**stellar structure**

**What is a star**



# stellare parameters

## What is a star





# parameters

initial mass:  $0.07 - 120 M_{\odot}$

$<$  (approximately)  $7 M_{\odot}$

$>$  (approximately)  $7 M_{\odot}$

low mass stars

massive stars

Luminosity:

$10^{-2} - 10^6 L_{\odot}$

Radius:

$0.01 - 1000 R_{\odot}$

temperatur at surface ( $\leftrightarrow T_{\text{eff}}$ ):

$3000 - 100000 \text{ K}$

temperatur in the core:

$10^6 - 5 \cdot 10^9 \text{ K}$

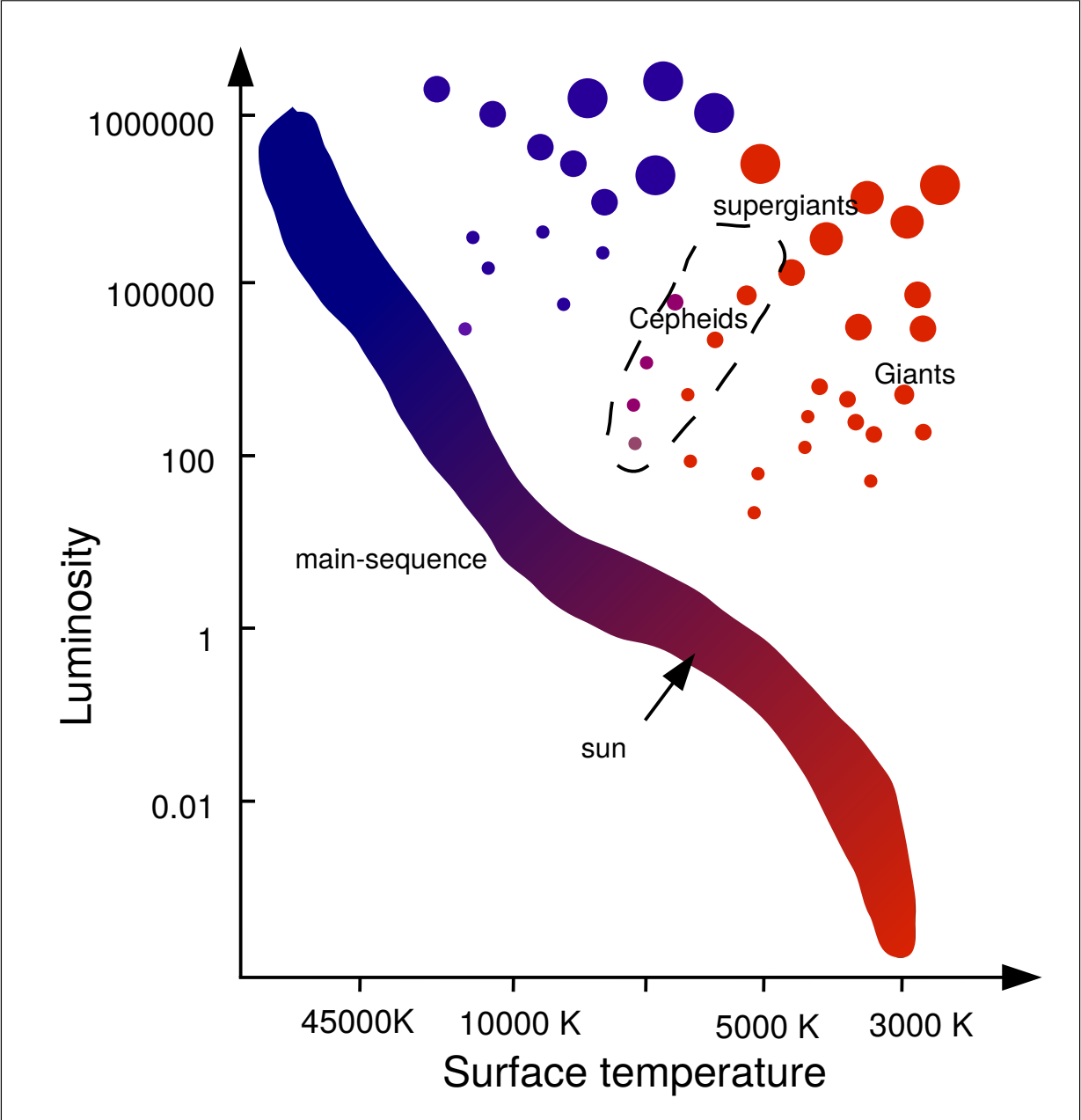
lifetime:

$10^6 - 10^{10} \text{ years}$

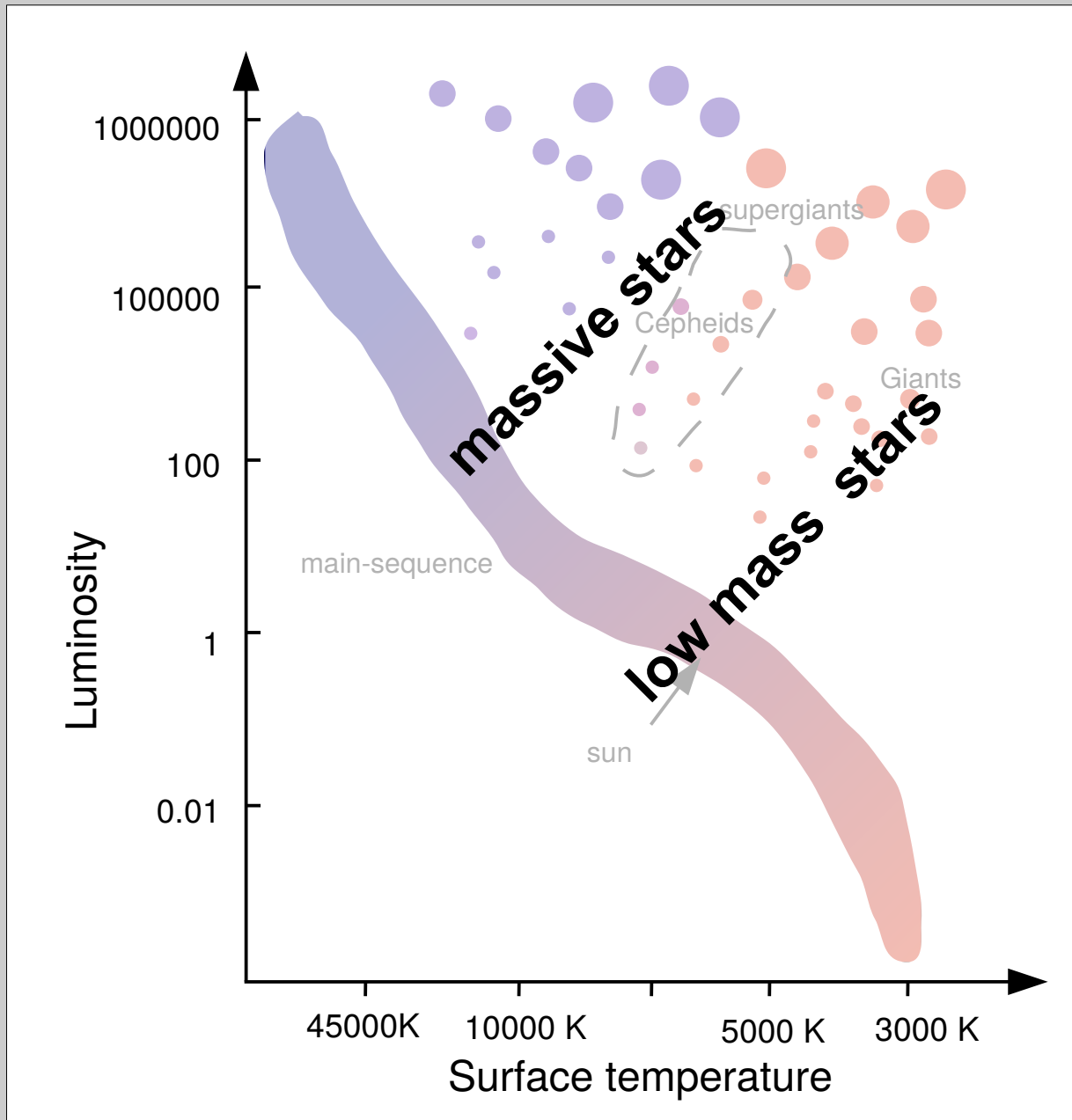


# Hertzprung-Russel-Diagramm

schematic



# Hertzsprung-Russel-Diagramm



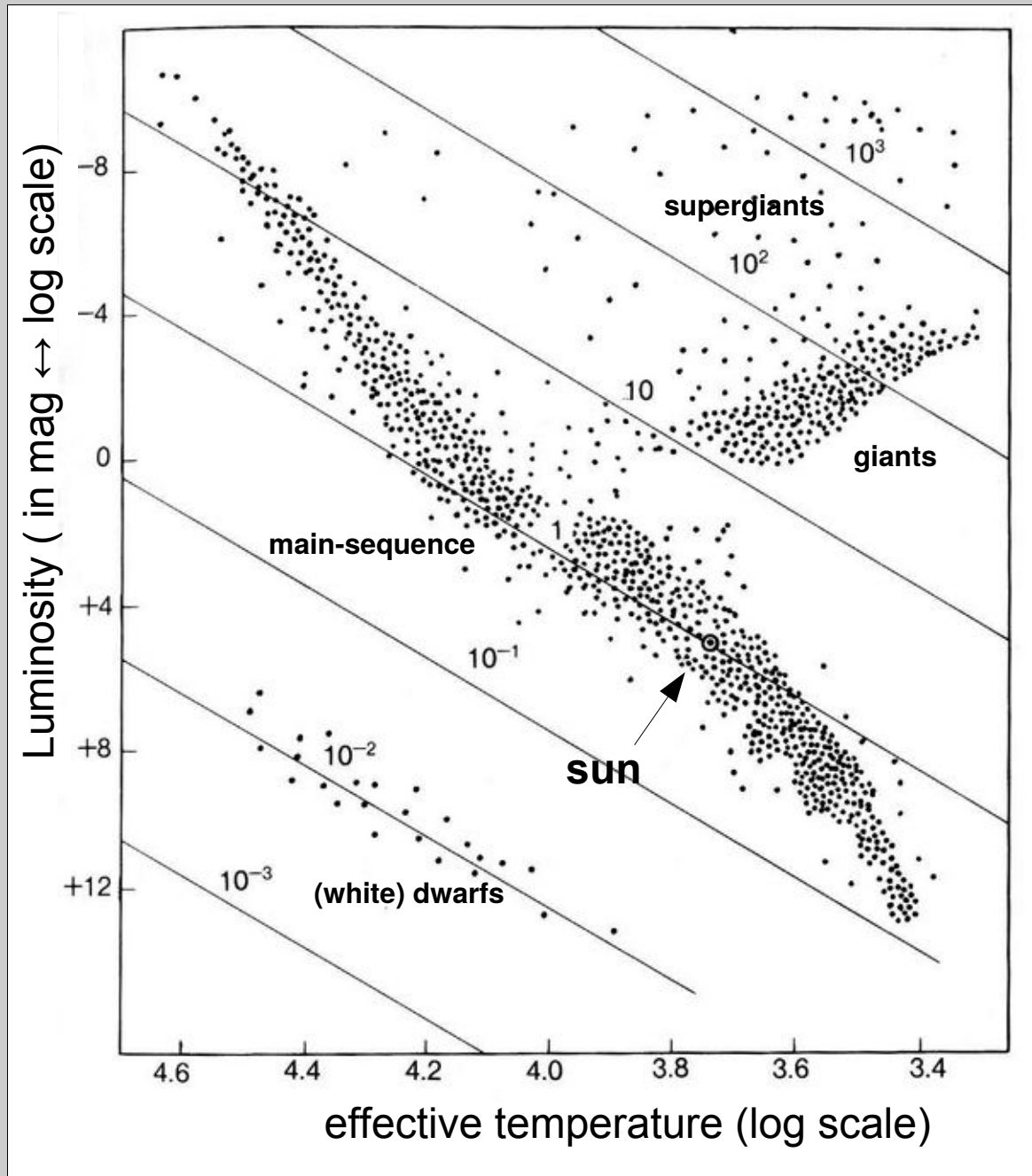
schematic

$\approx 7 M_{\odot}$   
**massive stars**

$\approx 7 M_{\odot}$   
**low mass stars**



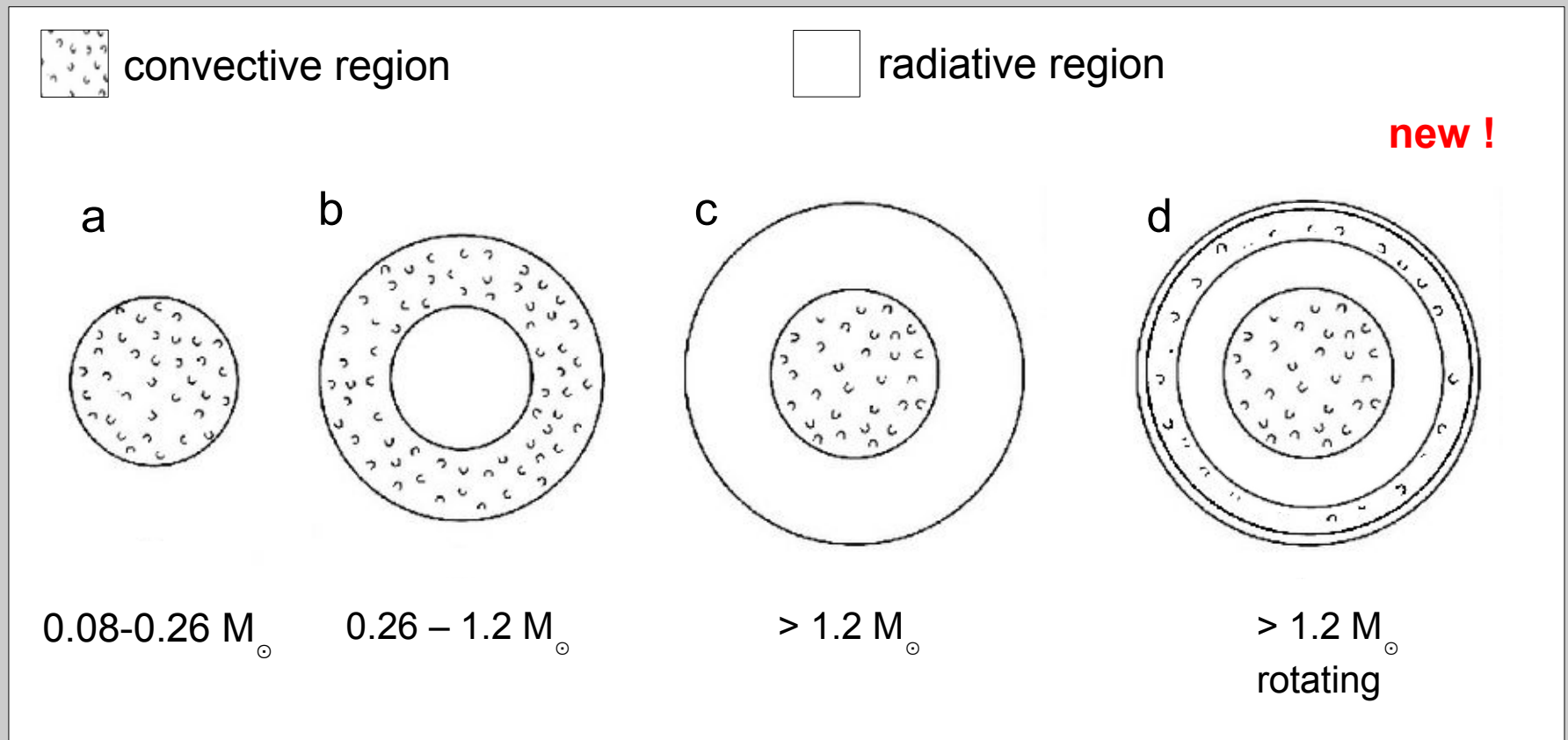
# Stellar radii in the HRD





# Innere Structure (main-sequence stars)

- a) low mass stars  $0.08-0.26 M_{\odot}$
- b) low mass stars up to  $1.2 M_{\odot}$
- c) low mass stars & massive stars above  $1.2 M_{\odot}$
- d) low mass stars & massive stars above  $1.2 M_{\odot}$  with **Rotation !**

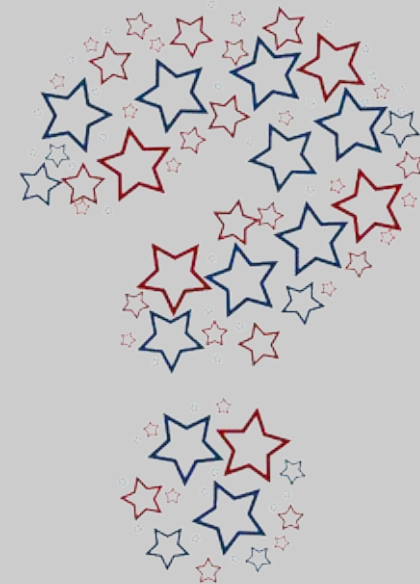


**star formation**

**stellare parameters**

**stellar structure**

**What is a star**



# stellar structure

## What is a star



# stellar structure



# stellar structure Vogt – Russell theorem



## Vogt-Russell theorem:



**the structure (radius, luminosity, etc.)  
of a star is uniquely  
determined by its mass ( $M$ ) and the amount of  
chemical elements  $\leftrightarrow$  metallicity ( $Z$ )**

(Vogt 1926, Russell 1927)

OKAY

# stellar structure Vogt – Russell theorem



## Vogt-Russell theorem:



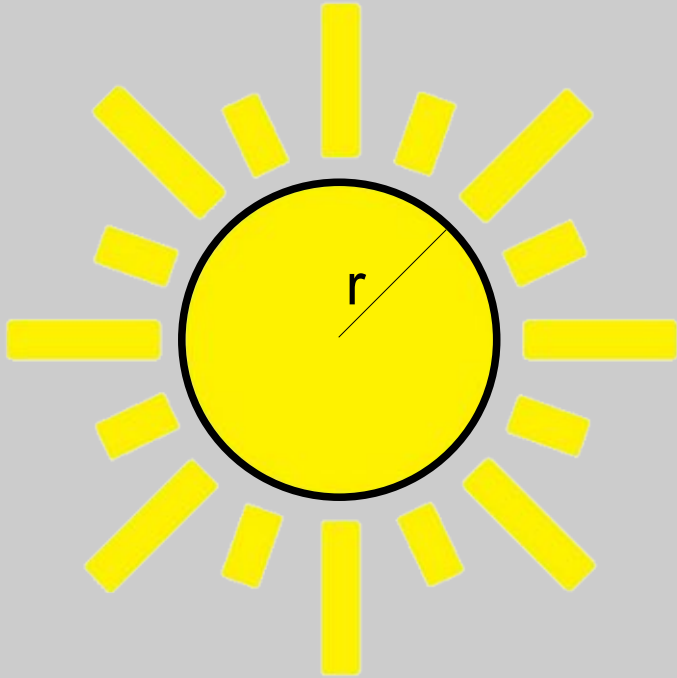
**the structure (radius, luminosity, etc.)  
of a star is uniquely  
determined by its mass ( $M$ ) and the amount of  
chemical elements  $\leftrightarrow$  metallicity ( $Z$ )**

(Vogt 1926, Russell 1927)

OKAY but UPDATES

**Mass loss  $\leftrightarrow$  stellar winds ( $dM/dt$ )**  
**rotation ( $v_{\text{rot}}$ ) und magnetic field ( $B$ ) are important !!!**

# mass continuity equation



$m :=$  mass     $A :=$  surface

assumption the star is a **sphere**

→ sounds trivial but isn't !

→ if a stars rotates fast it is an **ellipsoid** !

$$V = A dr \quad \text{sphere} \leftrightarrow A = 4\pi r^2$$

$$\rho = \frac{M}{V} \rightarrow \frac{dm}{4\pi r^2 dr}$$

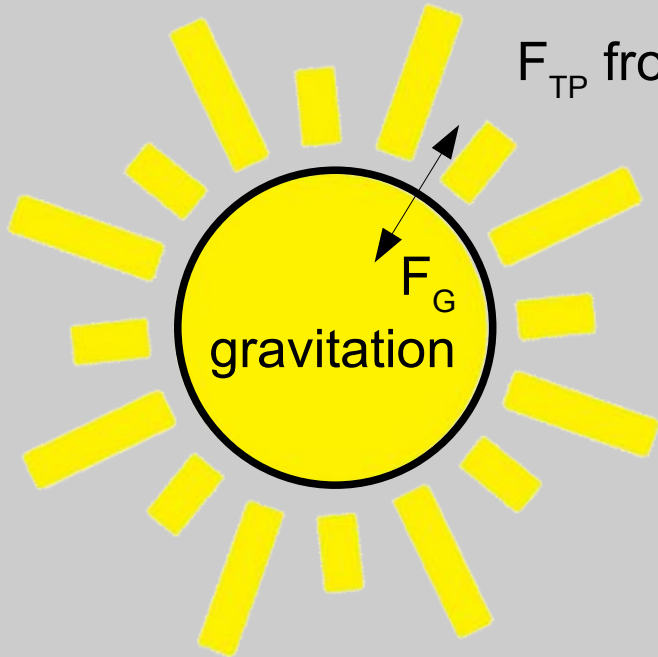
$$\frac{dm}{dr} = 4\pi r^2 \rho$$

more commonly used version

$$dm = 4\pi r^2 \rho dr$$



# hydrostatic equilibrium



$F_{TP}$  from thermal pressure

$$F_G = m \times g \rightarrow dF_G = G \frac{M dm}{r^2}$$

$$F_{TP} = A \times P \rightarrow dF_{TP} = AdP$$

$m :=$  mass  $P :=$  pressure  $A :=$  surface

mit density  $\rho = m/V = dm / A dr \rightarrow A = 1/\rho dm/dr$

balance of forces

$$dF = dm \frac{d^2r}{dt^2} = -G \frac{M dm}{r^2} - AdP \rightarrow G \frac{M dm}{r^2} = -\frac{1}{\rho} \frac{dm}{dr} dP$$

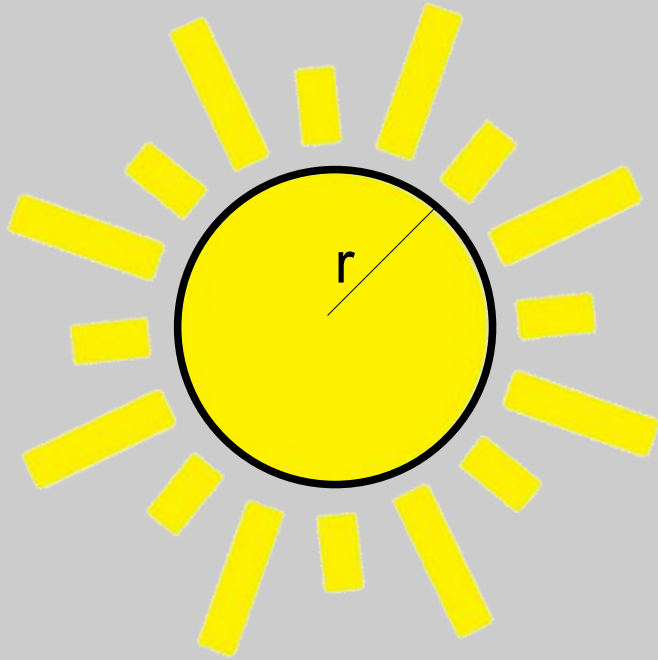
$$= 0$$

$$\rightarrow G \frac{M \cancel{dm}}{r^2} = -\frac{1}{\rho} \frac{\cancel{dm}}{dr} dP$$

$$g = \frac{G M}{r^2}$$

$$\frac{dP}{dr} = -\rho g$$

# energy equation



$L$  := Luminosity     $\varepsilon$  := energy generation rate

$$\rightarrow \varepsilon = \varepsilon_{\text{Nuklear}} + \varepsilon_{\text{Gravitation}}$$

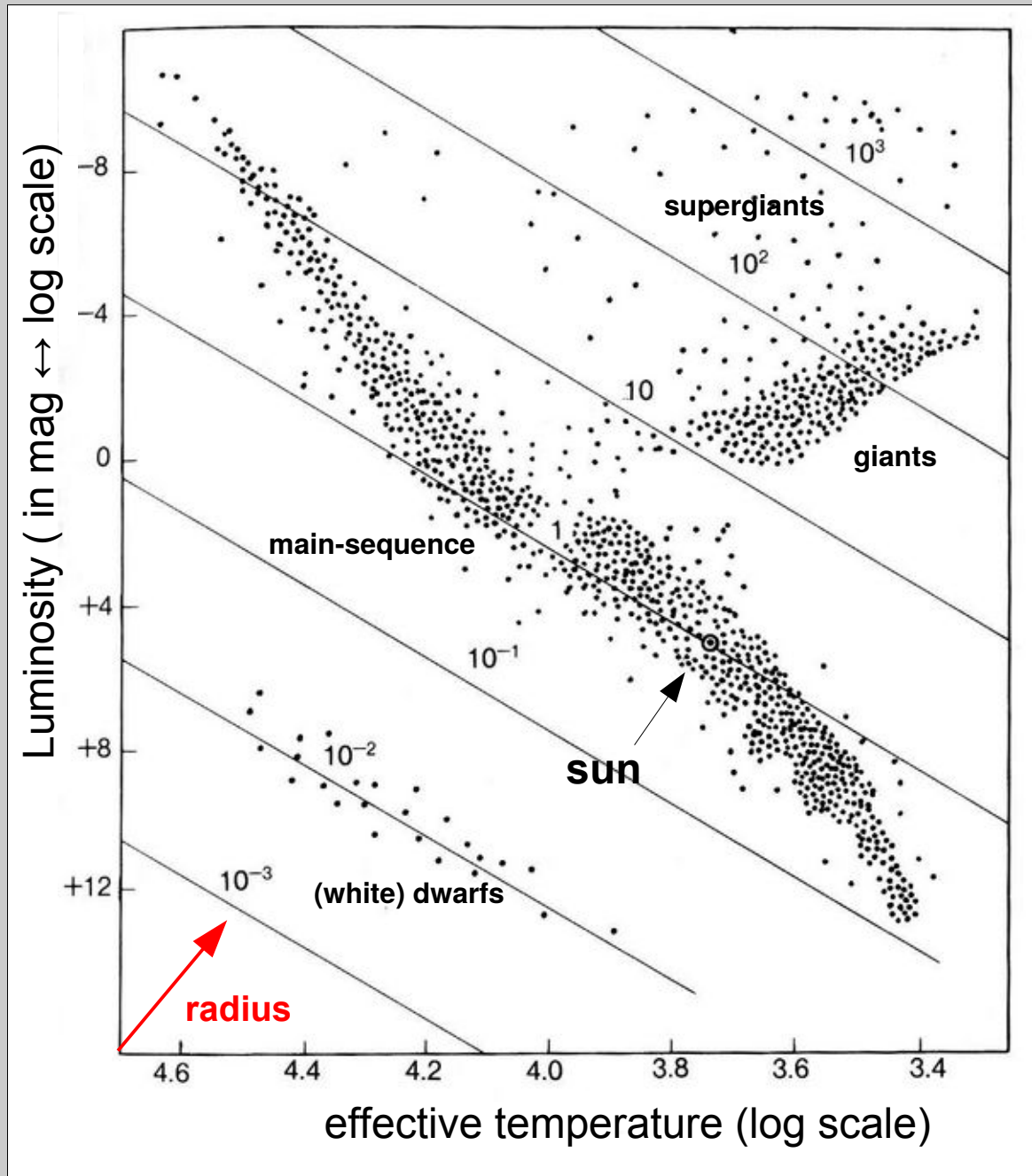
$$dL = \varepsilon dm$$

Using the mass continuity equation

$$dm = 4\pi r^2 \rho dr$$

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

# Stellar radii in the HRD



$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$

↔

$$dL = 4\pi r^2 \rho \epsilon dr$$

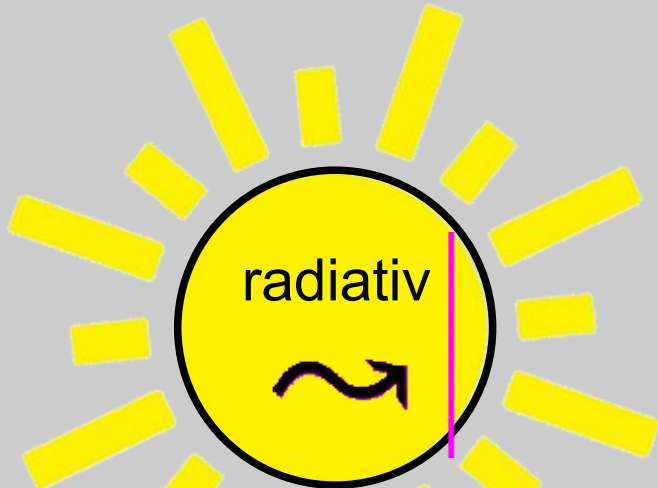
explains radius axis in HRD

# energy transport equation

$T$  := Temperature

$\gamma$ :  $c_p/c_v$  adiabatic coefficient

$\kappa$  = opacity



$$\frac{dT_{\text{rad}}}{dr} = -\frac{3}{4ac} \frac{\kappa \rho}{T^3} \frac{Lr}{4\pi r^2}$$

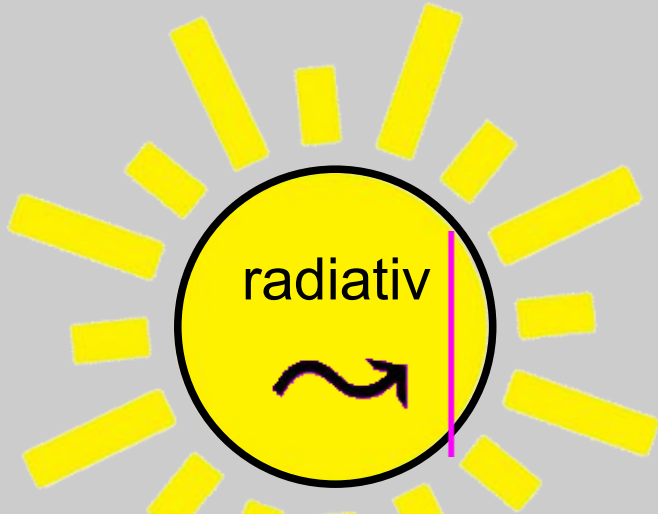
$$\frac{dT_{\text{con}}}{dr} = -\left(1 - \frac{1}{\gamma}\right) \frac{\mu m_H}{k} \frac{GM}{r^2}$$

sometimes written not as convective but **adiabatic**

$$\frac{dT_{\text{ad}}}{dr} = -\left(1 - \frac{1}{\gamma}\right) \frac{\mu m_H}{k} \frac{GM}{r^2}$$

# energy transport equation

"what dominates where ?"



$$\frac{dT_{\text{con}}}{dr} < \frac{dT_{\text{rad}}}{dr} \rightarrow \text{radiativ transport}$$



$$\frac{dT_{\text{con}}}{dr} > \frac{dT_{\text{rad}}}{dr} \rightarrow \text{transport by convection}$$

# stellar structure equations

mass continuity equation

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho g$$

energy equation

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$

energy transport equation

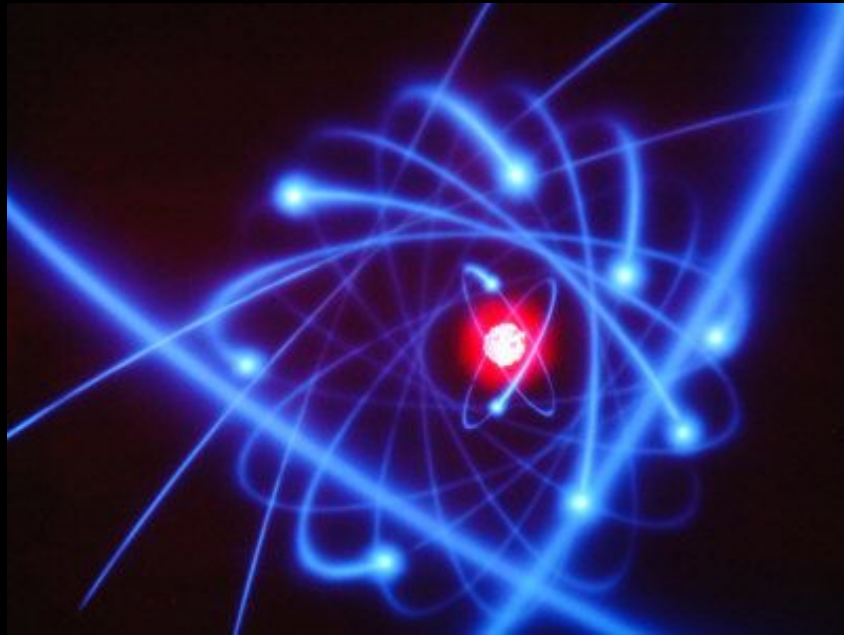
$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa \rho}{T^3} \frac{Lr}{4\pi r^2}$$
$$\frac{dT}{dr} = -\left(1 - \frac{1}{\gamma}\right) \frac{\mu m_H}{k} \frac{GM}{r^2}$$



**energy**



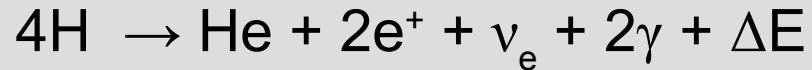
**generation**





# energy generation – nuclear burning process

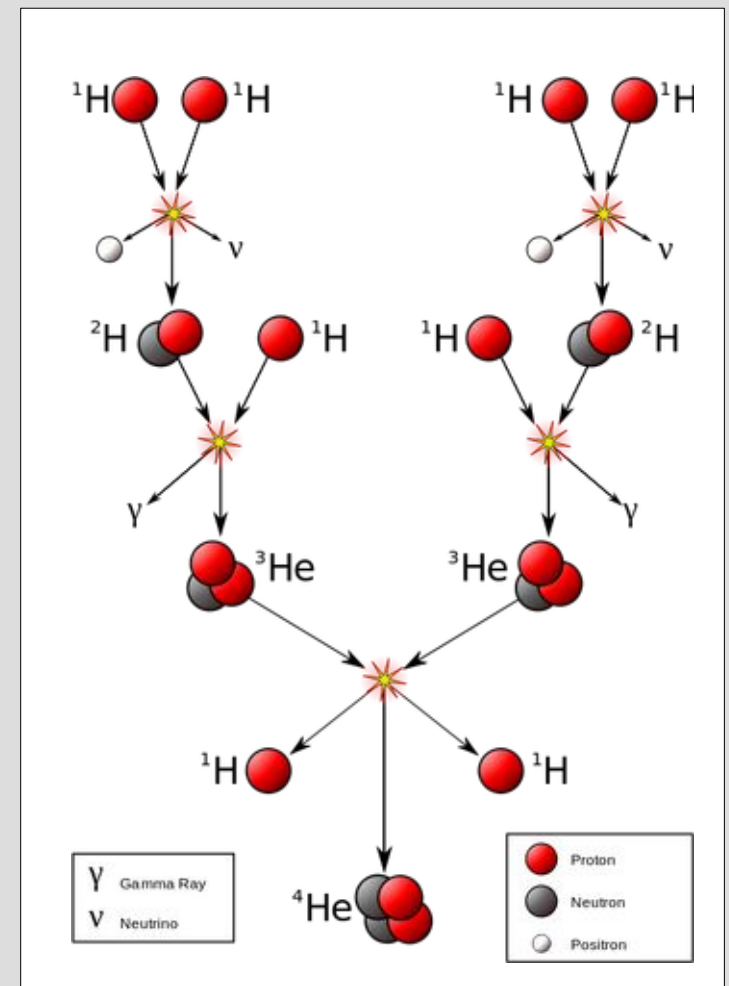
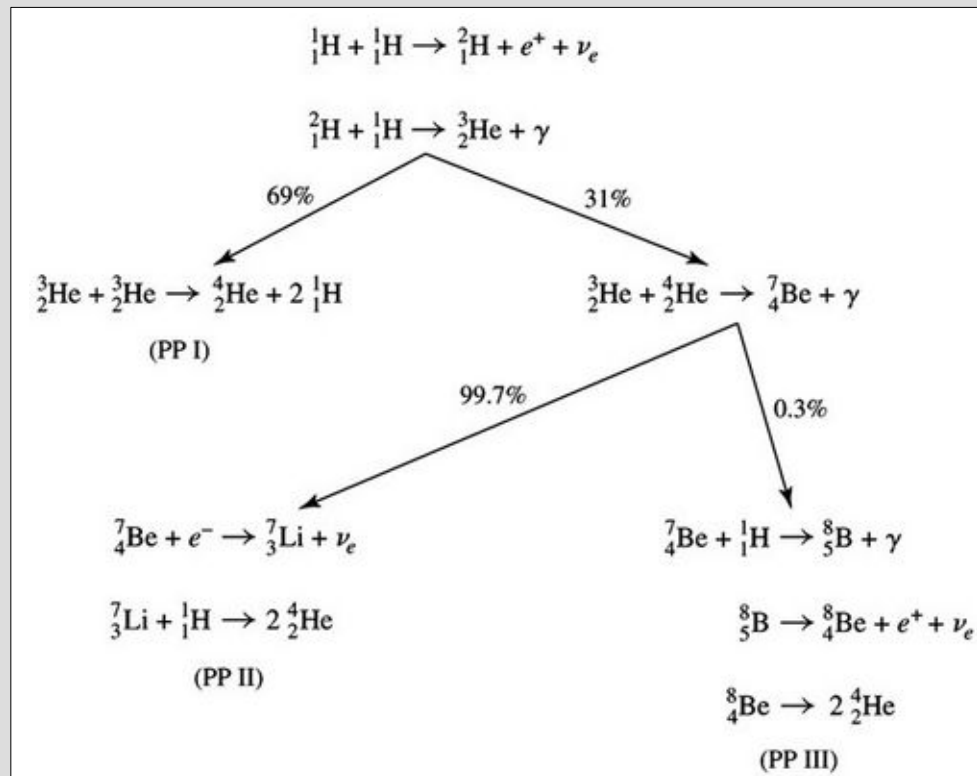
## Hydrogen burning the PP-chain(s)



$$\Delta E = 26.2 \text{ MeV}$$

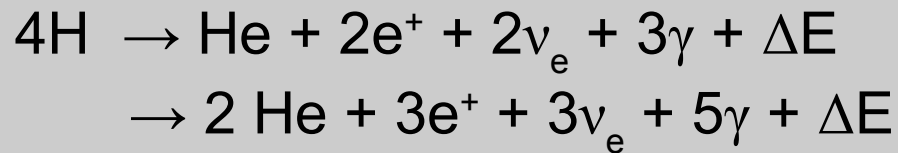
$$\varepsilon \propto T^{4-6}$$

$$T \approx 1.5 \cdot 10^7 \text{ K}$$



# energy generation – nuclear burning process

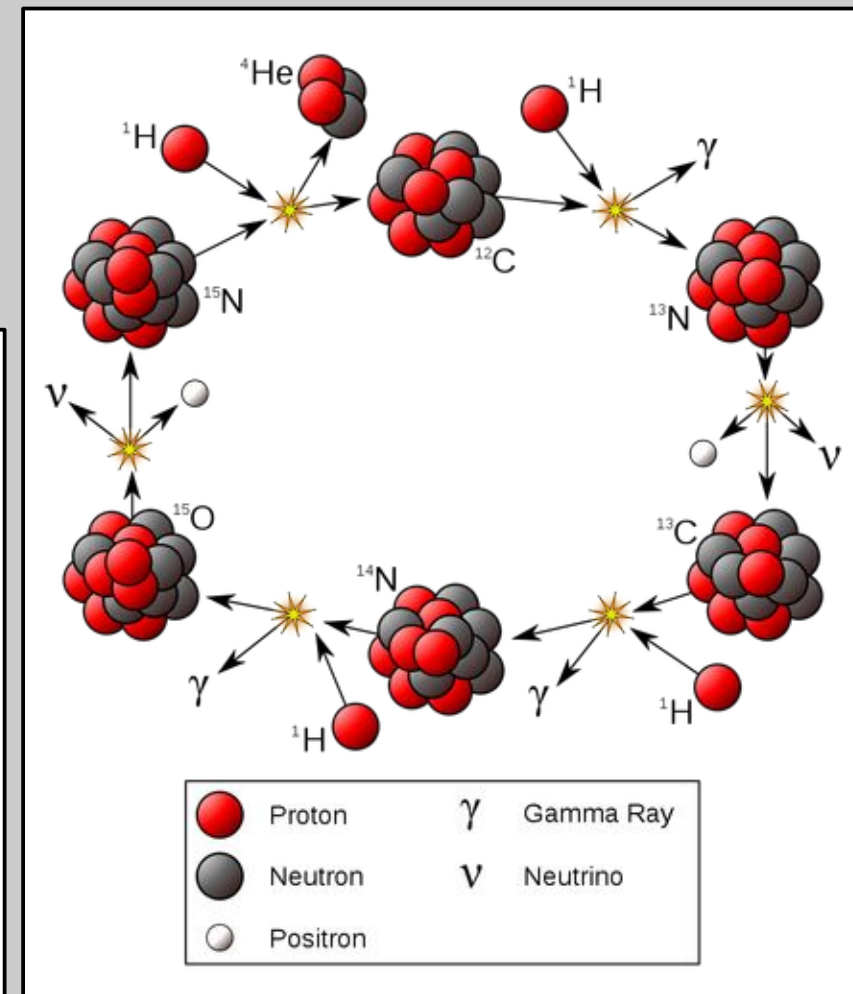
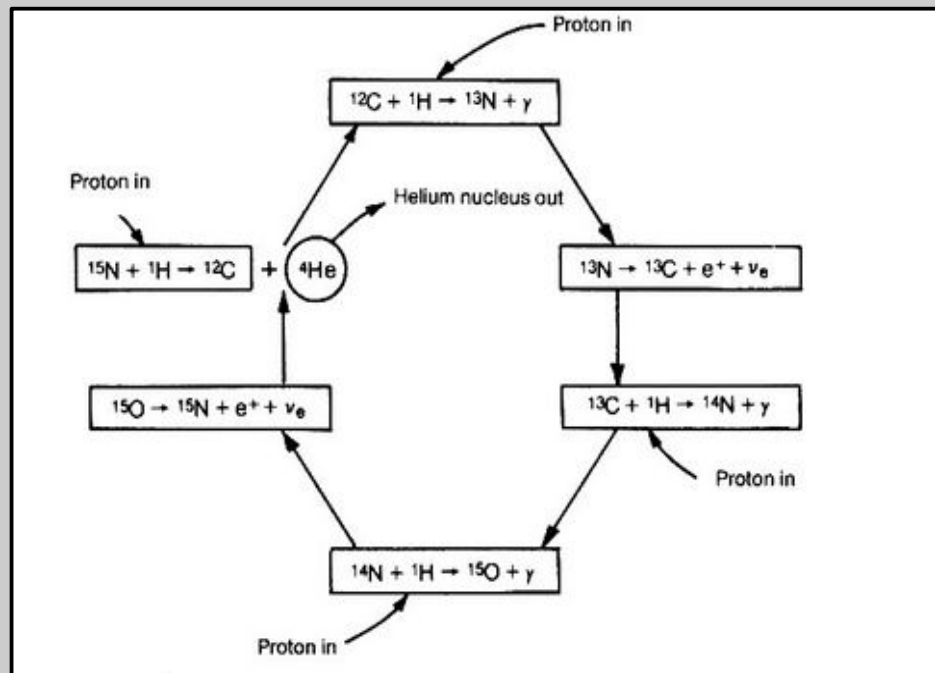
## Hydrogen burning the CNO-cycle



$$\Delta E = 25 \text{ MeV}$$

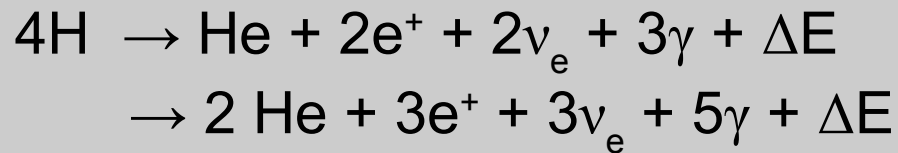
$$\varepsilon \propto T^{15-16}$$

$$T \geq 1.5 \cdot 10^7 \text{ K}$$



# energy generation – nuclear burning process

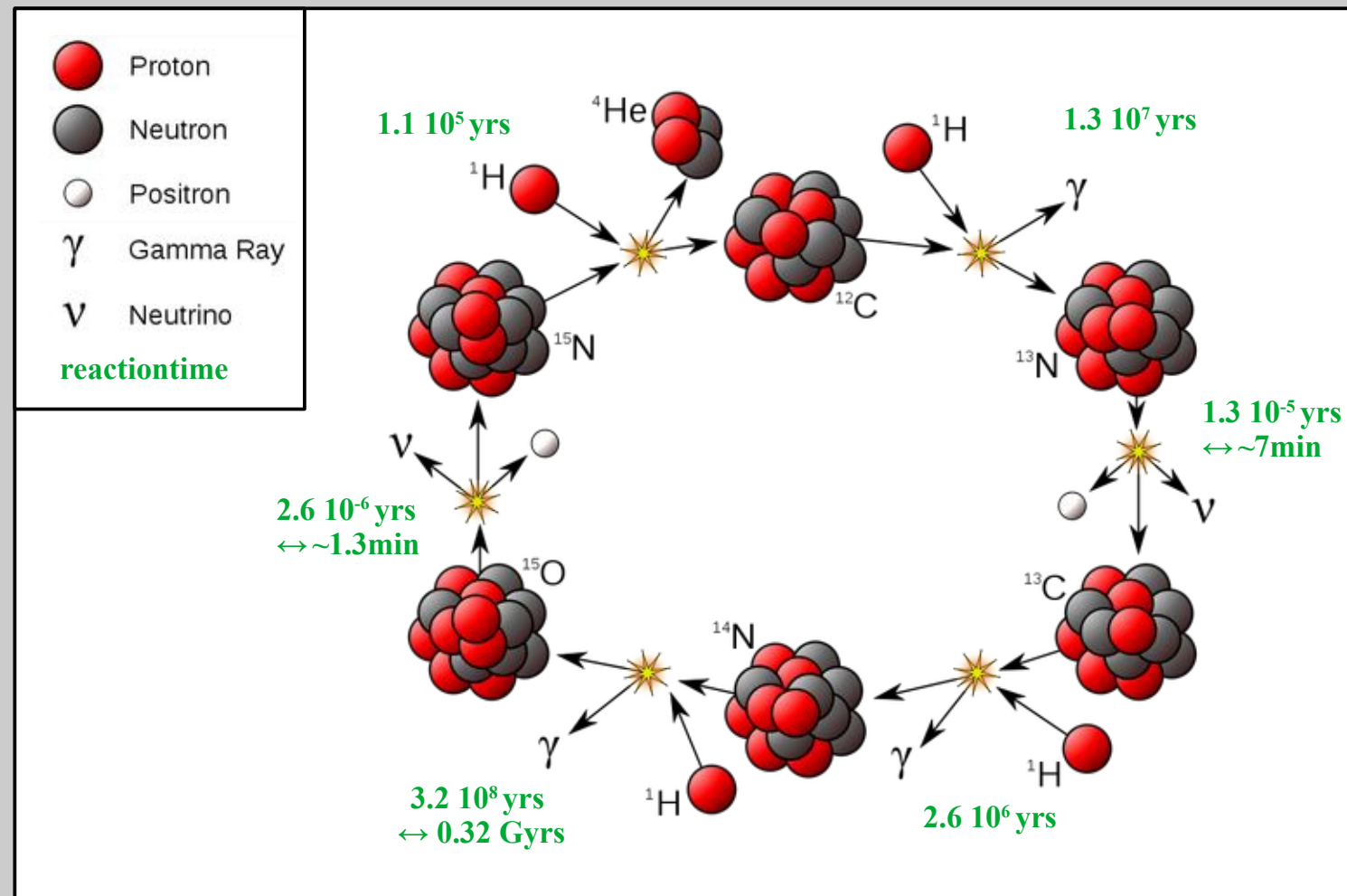
## Hydrogen burning the CNO-cycle



$$\Delta E = 25 \text{ MeV}$$

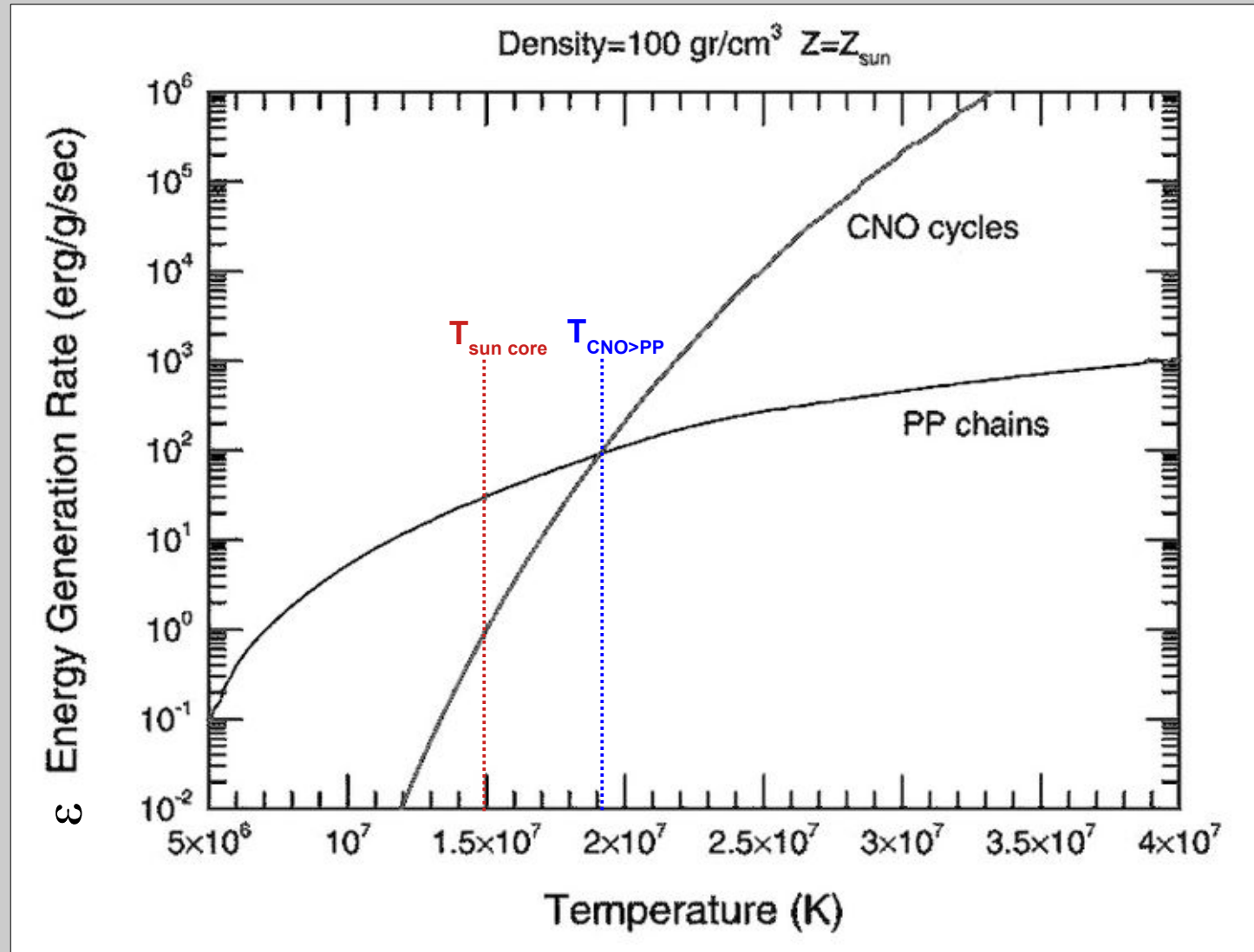
$$\varepsilon \propto T^{15-16}$$

$$T \geq 1.5 \cdot 10^7 \text{ K}$$



# CNO cycle versus pp-chain

Efficiency of the pp and CNO process



# burning phases and stellar lifetimes

The **duration** of the H **burning phase**  $\leftrightarrow$  the by far longest burning process (= MS lifetime) for all stars and good total lifetime estimate.

It is **shorter** for a higher stellar **mass**.

$$\tau_{\text{MS}} \approx 10^{10} \text{ years} \left[ \frac{M}{M_{\odot}} \right] \left[ \frac{L_{\odot}}{L} \right] = 10^{10} \text{ years} \left[ \frac{M}{M_{\odot}} \right]^{-2.5}$$

H burning  $\leftrightarrow$  MS lifetime

1 $M_{\odot}$	1 $10^{10}$ years
10 $M_{\odot}$	3 $10^7$ years
50 $M_{\odot}$	6 $10^5$ years
80 $M_{\odot}$	2 $10^5$ years

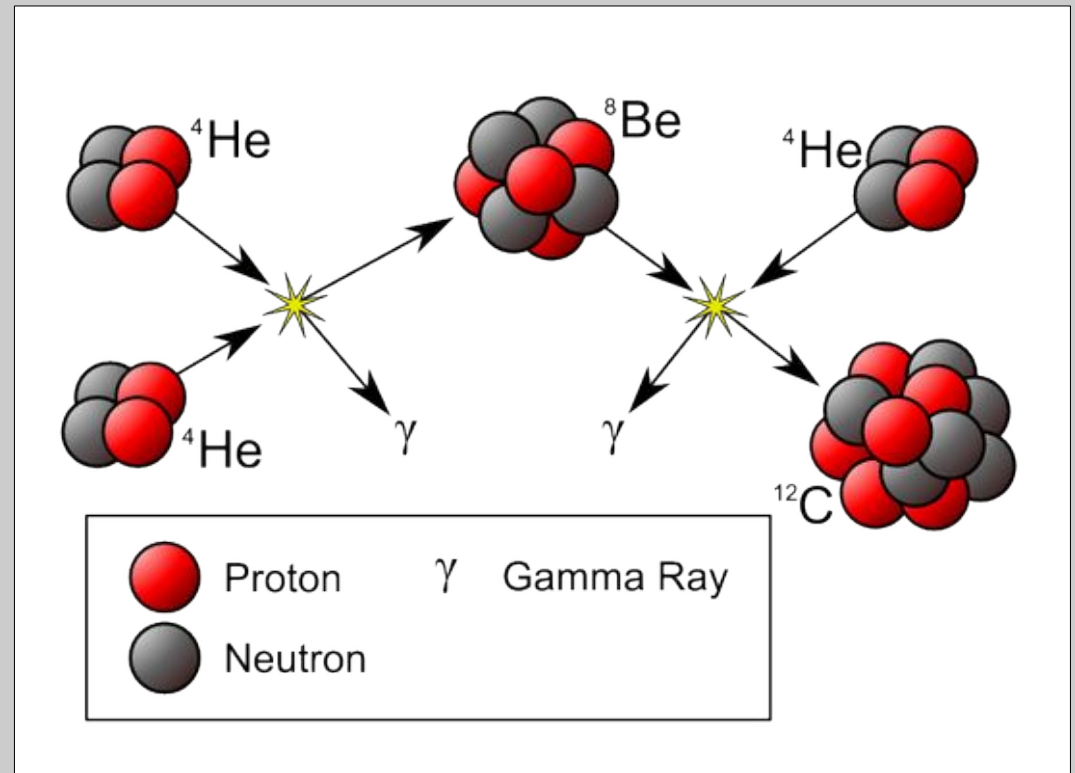
# Helium burning

## Helium-burning or Triple- $\alpha$



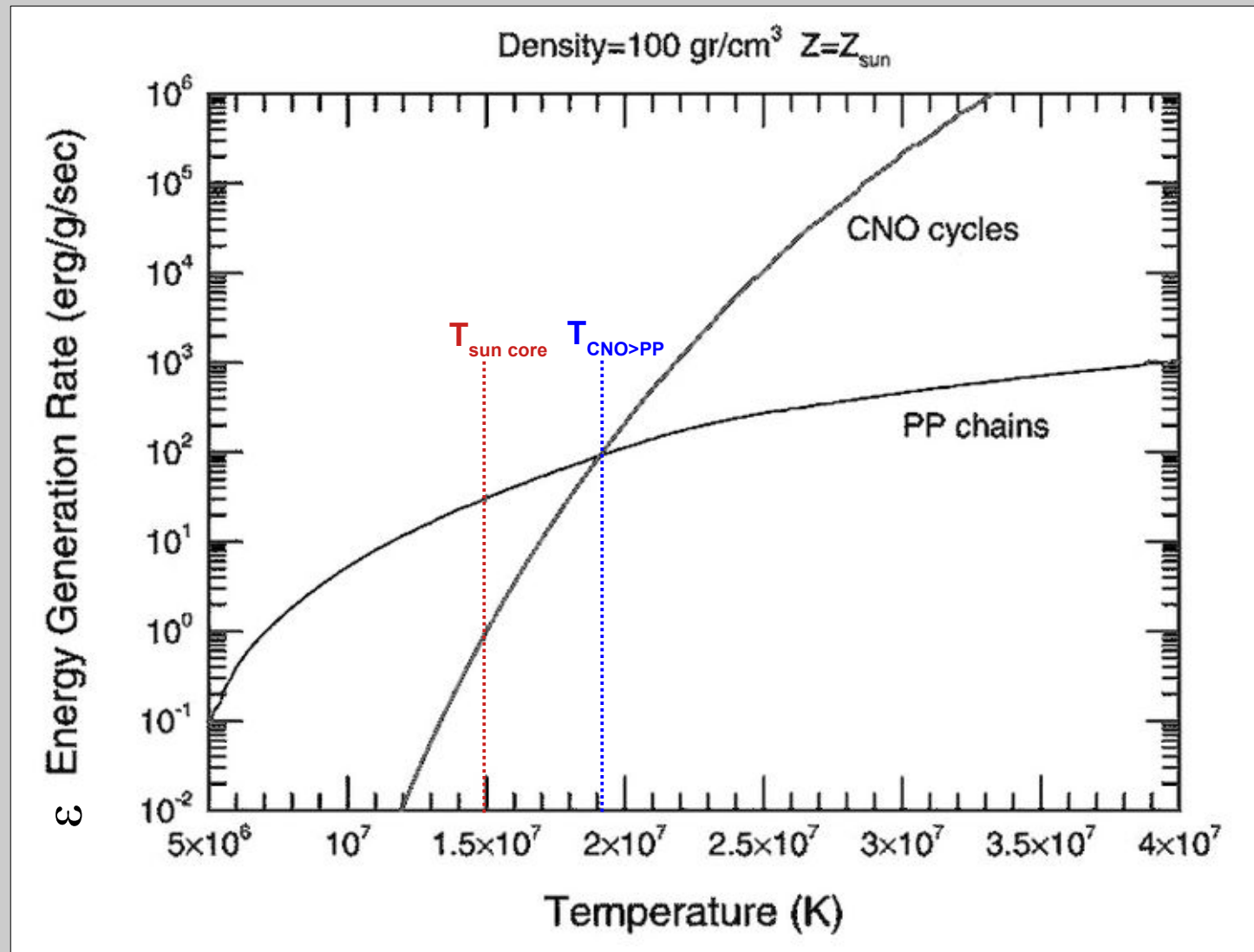
$$\Delta E = 7.3 \text{ MeV}$$
$$T > 1.5 \cdot 10^8 \text{ K}$$

$$\varepsilon \propto T^{30-40}$$



# CNO cycle versus pp-chain

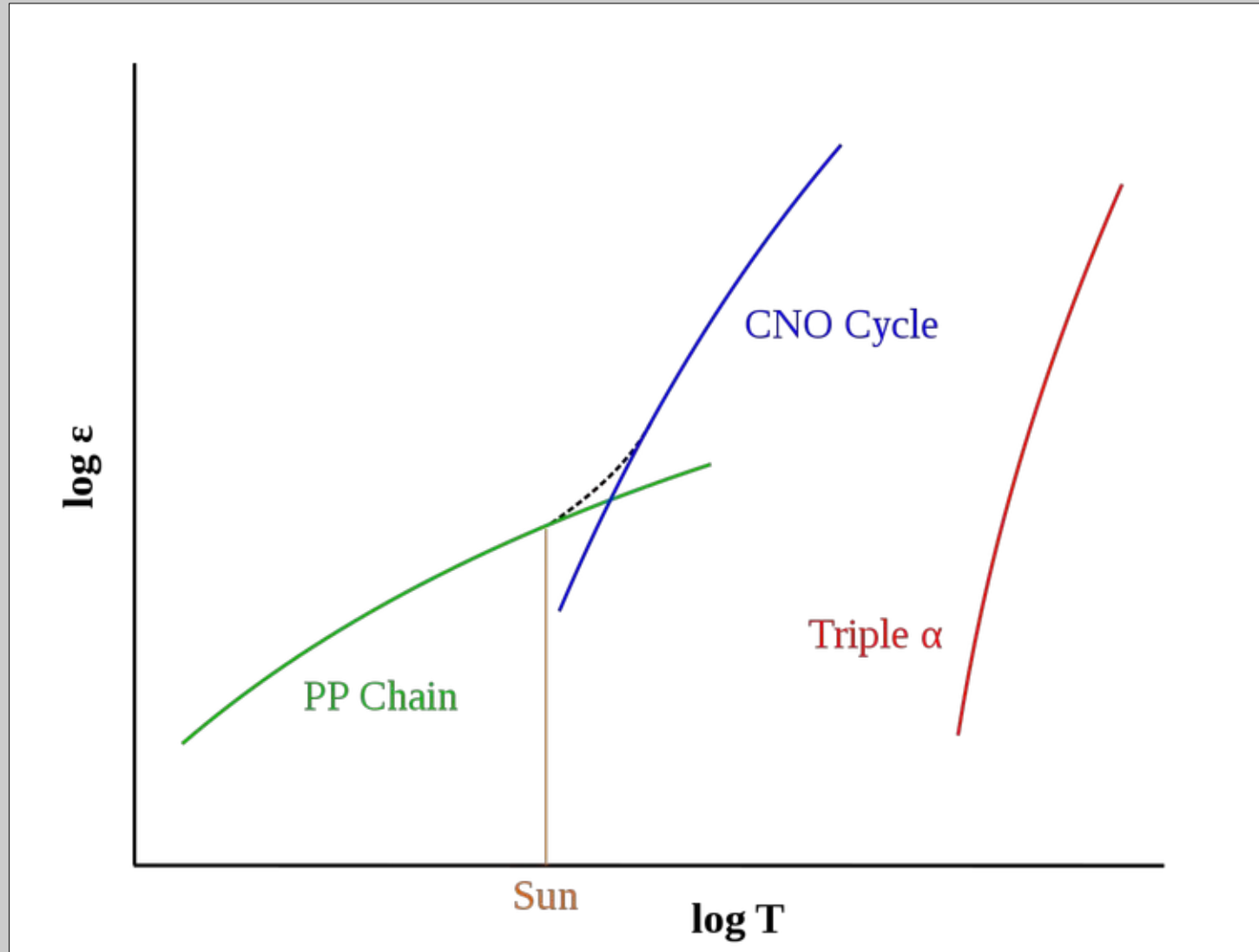
Efficiency of the pp and CNO





# PP • CNO • He $\leftrightarrow$ Triple $\alpha$ burning

Efficiency of the pp CNO and Triple  $\alpha$  process



# higher burning phases

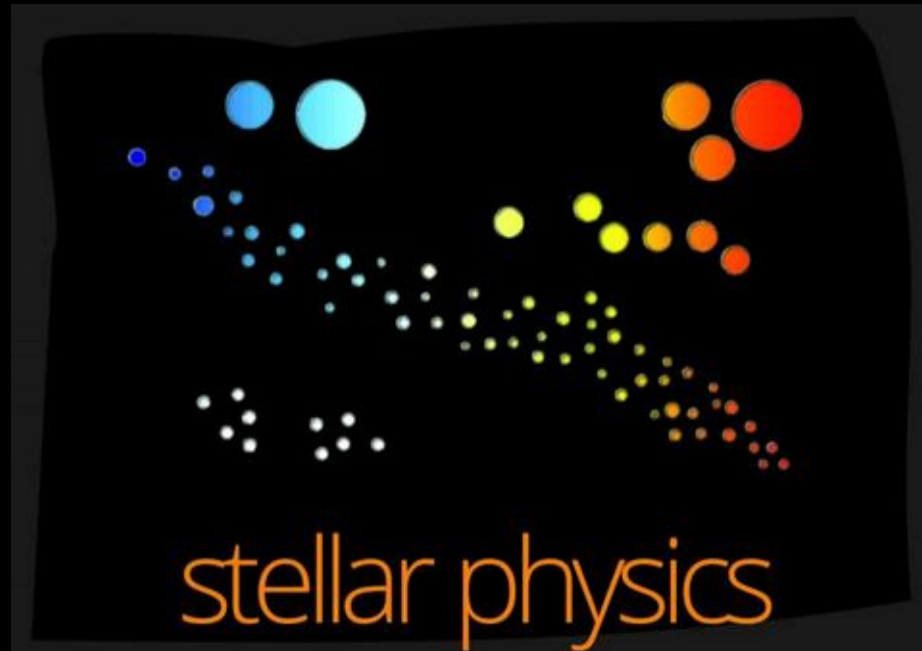
**Carbon-burning**  $C + He \rightarrow O + \gamma + \Delta E$   $\Delta E = 7.2 \text{ MeV}$

**Oxygen-burning**  $O + He \rightarrow Ne + \gamma + \Delta E$   $\Delta E = 4.6 \text{ MeV}$

*Massive Stars  
only!*

**Neon and Silicon burnings follow** and create elements up to **Iron**.

for **more** details **see** the **lecture on**



# Duration of burning phases

The **duration** of the each **burning phase**  
is  
**shorter** for **higher** elements  
and  
**faster** for stars with **higher mass**

**Table 3 Lifetimes of core burning phases**

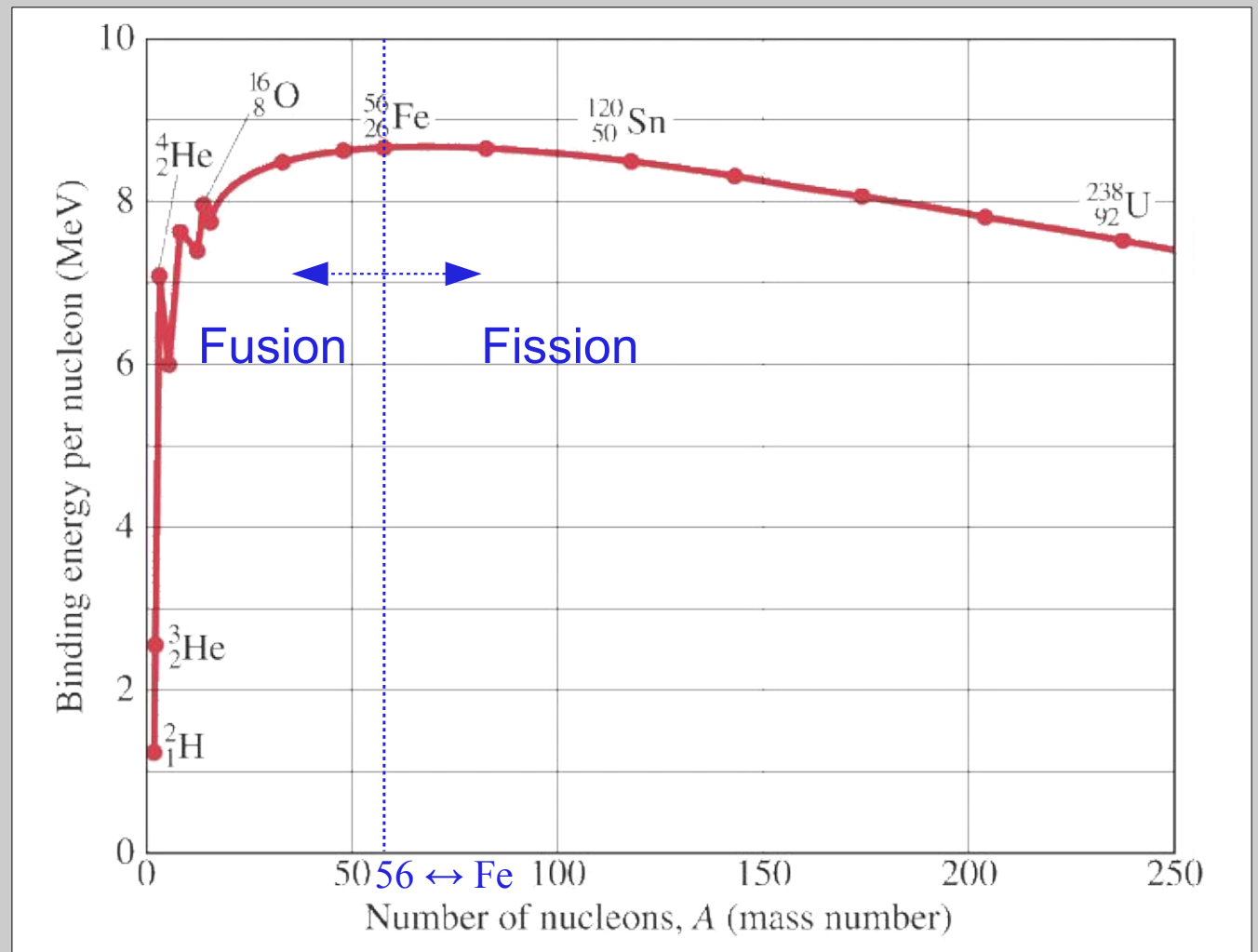
burning phase	time in years
H burning	$10^{5-9}$ years
He burning	$10^6$ years
O burning	300 years
Ne burning	200 days
Si burning	2 days

# Fusion versus Fission

Neon and Silicon burnings follow and create elements up to Iron.

Iron has the highest binding energy per nucleon

→ up to iron energy is set free in **Fusion** processes beyond in **Fission** (spallation)



# So what about gold und silver etc. ?



If Fusion burning stops with Iron how  
are elements with a higher atomic number formed ?



# So what about gold und silver etc. ?



Goldwaschen  
in Deutschland



Die ergiebigsten  
Gewässer, die  
besten Regionen!

The logo for GOLD.DE, featuring a stylized gold bar icon followed by the text "GOLD.DE" in a bold, sans-serif font.

# So what about gold und silver etc. ?



If Fusion burning stops with Iron how are elements with a higher atomic number formed ?

Instead of nuclear fusion **capturing** a **proton** or **neutron** leads to the formation new elements

There are 3 major process

- **p-Prozess** ↔ proton capture
- **s-Prozess** ↔ slow (s) neutron capture
- **r-Prozess** ↔ rapid (r) neutron capture



# p-process

p-process → **p** from **proton**

- by capturing **protons** the nucleus has a higher number of protons → new element
- in most cases a **photodesintegration** ↔ **γ-process** follows  
(Example  $\text{Ne} + \gamma \rightarrow \text{O} + \text{He}$ )
- the higher the element ↔ the higher is the Coulomb Wall and the process becomes more difficult
- so far unusual elements found with a maxima at  $^{92}\text{Molybdenum}$  und  $^{144}\text{Samarium}$

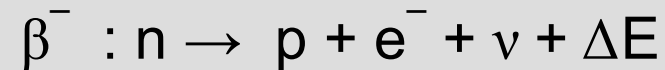
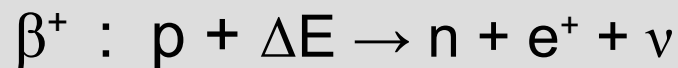
Occurs in: **Oxygen burning & Supernovae**



# r-process

r-process → **r** from **rapid**

- by capturing **neutrons** the nucleus has a higher number of neutrons / protons → new element/isotope
- competing process here is a  $\beta$  decay



if

$$T_{\text{n-capture}} \ll T_{\beta\text{-decay}}$$

**no  $\beta$  decay** occurs and the nucleus captures another **neutron**  
↔ **atoms with higher neutron and mass number!**

- known mass number  $A=130-190$

r process elements  $^{80}\text{Se}, ^{81}\text{Br}, ^{84}\text{Kr}, ^{128}\text{Te}, ^{130}\text{Te}, ^{127}\text{I}, ^{192}\text{Os}, ^{193}\text{Ir}, ^{196}\text{Pt}, ^{198}\text{Pt}$

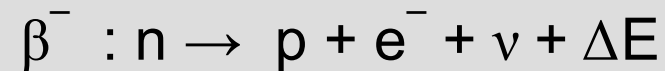
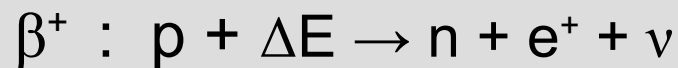
Occurs in: **Supernovae**



# s-process

s-process → **s** from **slow**

- by capturing **neutrons** the nucleus has a higher number of neutrons / protons → new element/isotope
- competing process here is a  $\beta$  decay



if

$$T_{\text{n-capture}} \ll T_{\beta\text{-decay}}$$

**a  $\beta$  decay occurs before another neutron is captured  $\leftrightarrow$  n converted to p atoms with higher proton and mass number!**

- known mass number  $A=$ bis 210  
s process elements ( $^{88}\text{Sr}, ^{138}\text{Ba}, ^{208}\text{Pb}, ^{89}\text{Y}, ^{90}\text{Zr}, ^{139}\text{La}$ )

Occurs in: **Red Giants, Red Supergiants & AGB** in shell burning region



The logo for the game "Stellar Evolution" is centered at the top. It features a stylized spaceship on the left, with a yellow beam of light extending from its front towards the right. The word "STELLAR" is written in a large, blue, blocky font with a white outline, positioned above the word "EVOLUTION", which is also in the same blue, blocky font with a white outline. The background is a dark blue space filled with numerous small white stars.

# STELLAR EVOLUTION

**NEW GAME**

Start the game from the beginning

**PASSWORD**

Enter a password to continue

**QUIT GAME**

Return to the Desktop

# Stellar evolution – models

Solve the stellar structure equations in time

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\rho g$$

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa \rho}{T^3} \frac{Lr}{4\pi r^2}$$

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

$$\frac{dT}{dr} = -\left(1 - \frac{1}{\gamma}\right) \frac{\mu m_H}{k} \frac{GM}{r^2}$$

a) boundary conditions

für  $r \rightarrow 0$       $M \rightarrow 0$  ,  $L \rightarrow 0$

für  $r \rightarrow R_*$       $P \rightarrow 0$  ,  $\rho \rightarrow 0$

b) use material functionen

$P(\rho, T, Z)$      **equation of state** (most cases ideal Gas)

$\kappa(\rho, T, Z)$      **opacity** by free free and  $e^-$  scattering

$\varepsilon(\rho, T, Z)$      **energy production** (most cases power law)

# Input – Output

**Input**

Stellar structure equations  
+  
material functionen  
+  
energy production



radius, temperature, luminosity  
of a stars as function a of time



Stellar evolution

**Output**

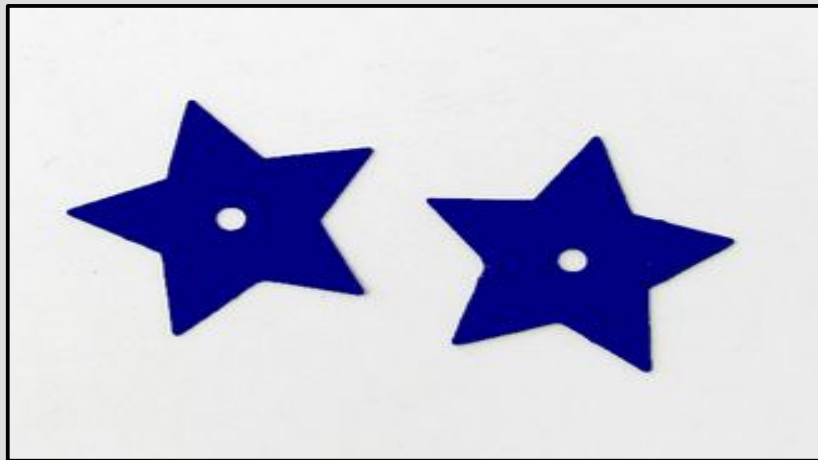
# Stellar evolution – models

" Solving the equation one integrates from the inside out as well as from the outside in hoping that the solutions meet somewhere and are steady...

In our first try in the 1970ies we always got solutions that had

**STARS WITH CENTRAL HOLES !!! "**

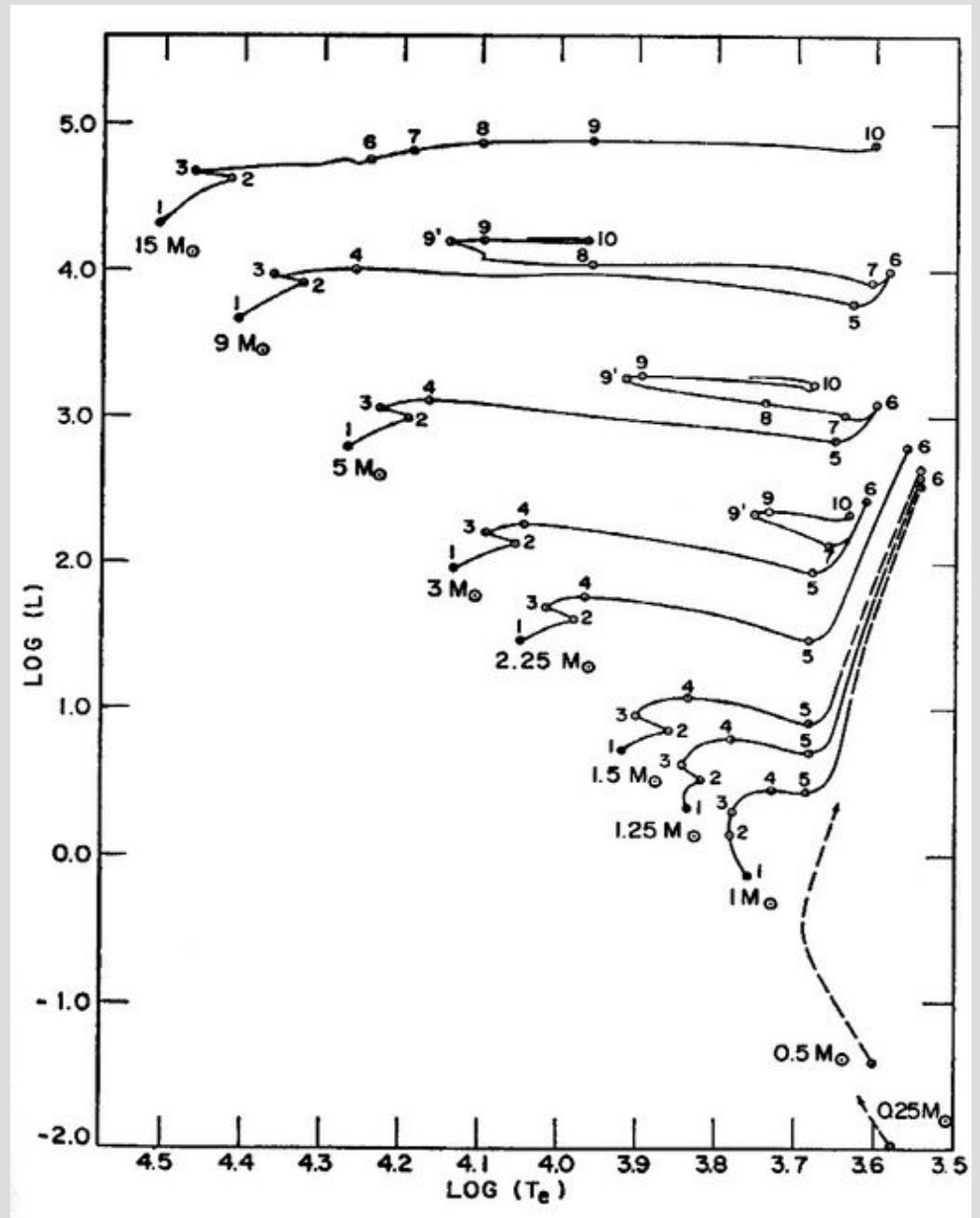
Prof. Rudolf Kippenhahn





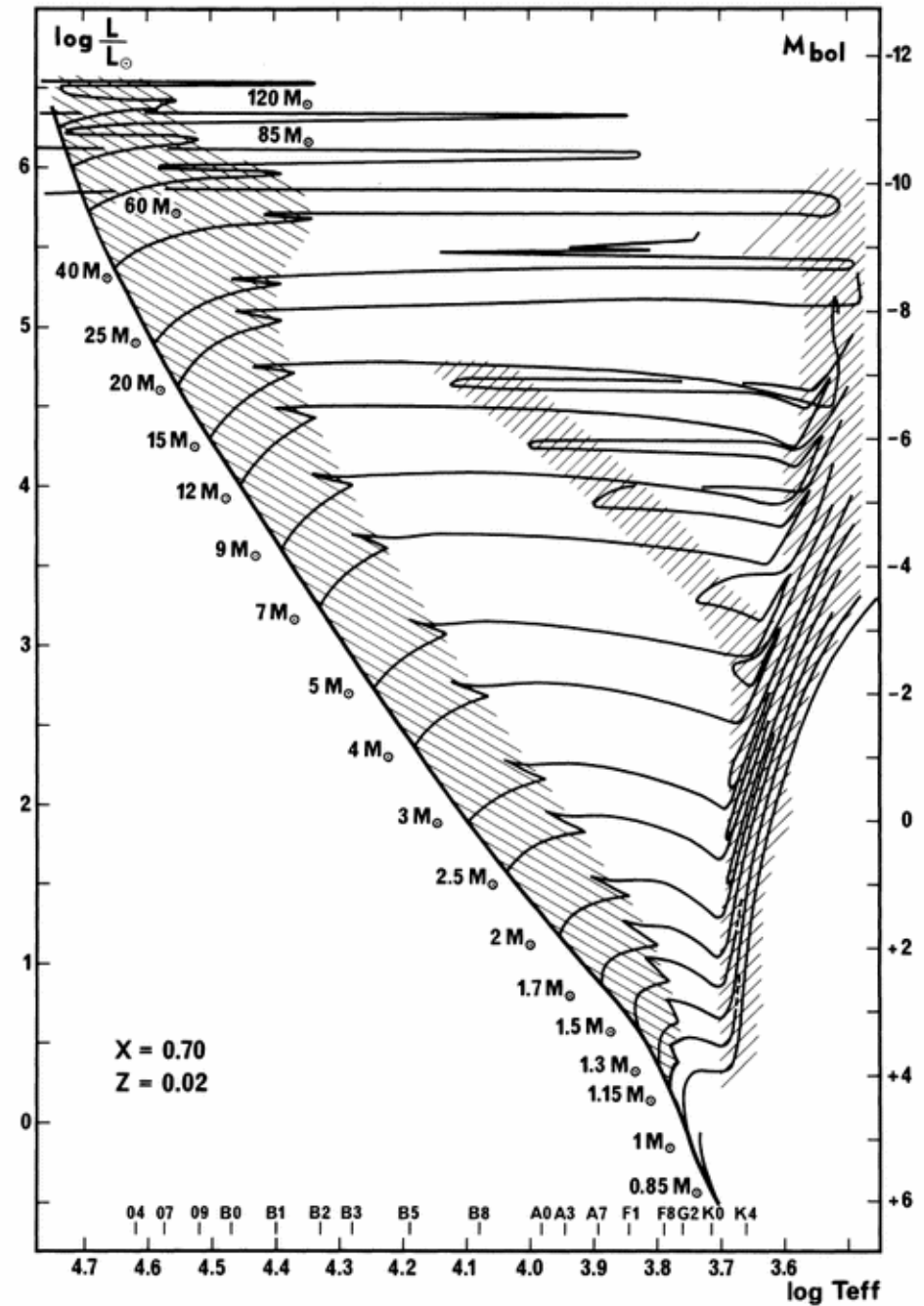
# Stellar evolution – models

At the beginning 1965-1980



# Stellar evolution – models

early 1980ties →



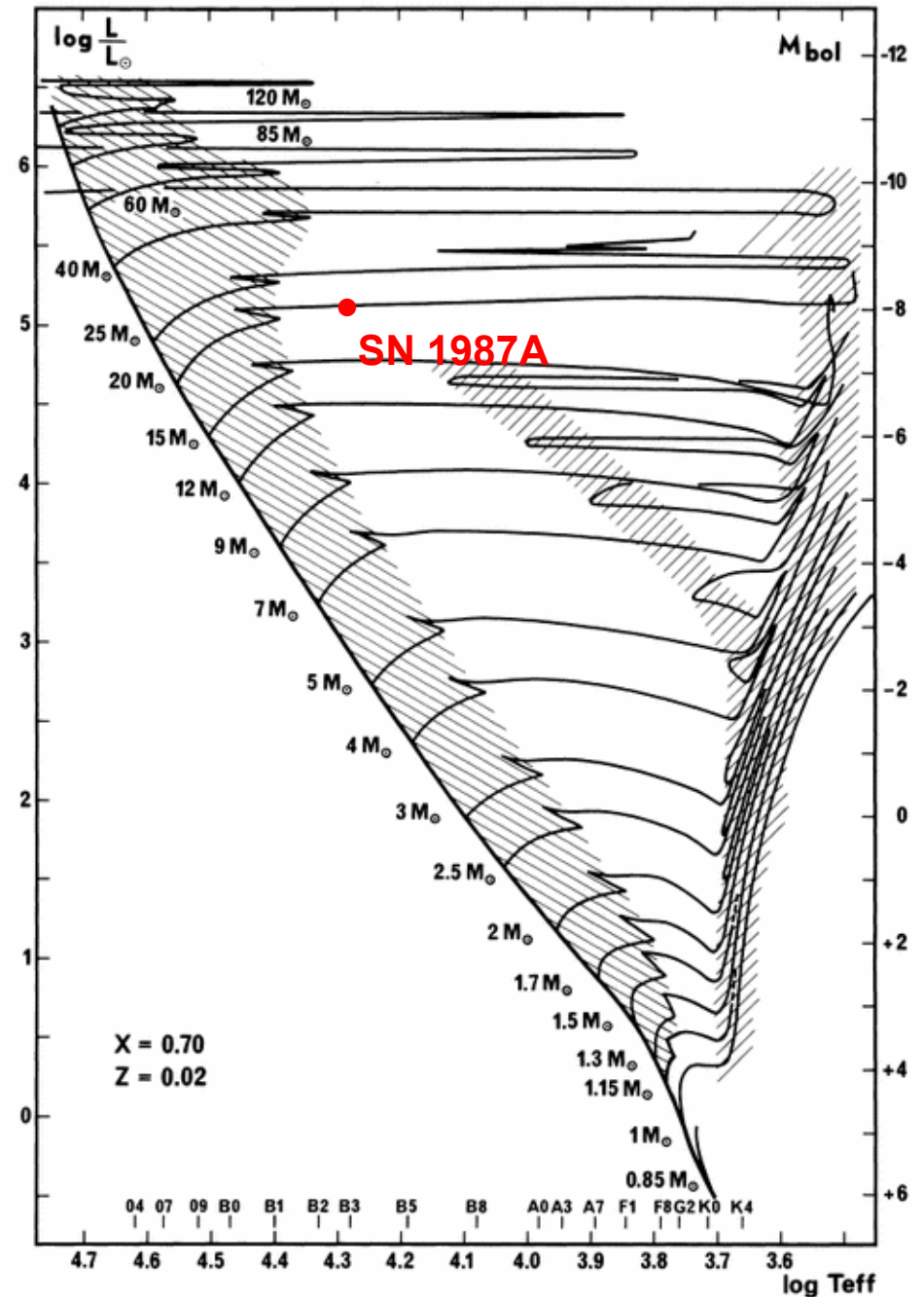
# Stellar evolution – models

early 1980ties →

## Problem SN 1987A



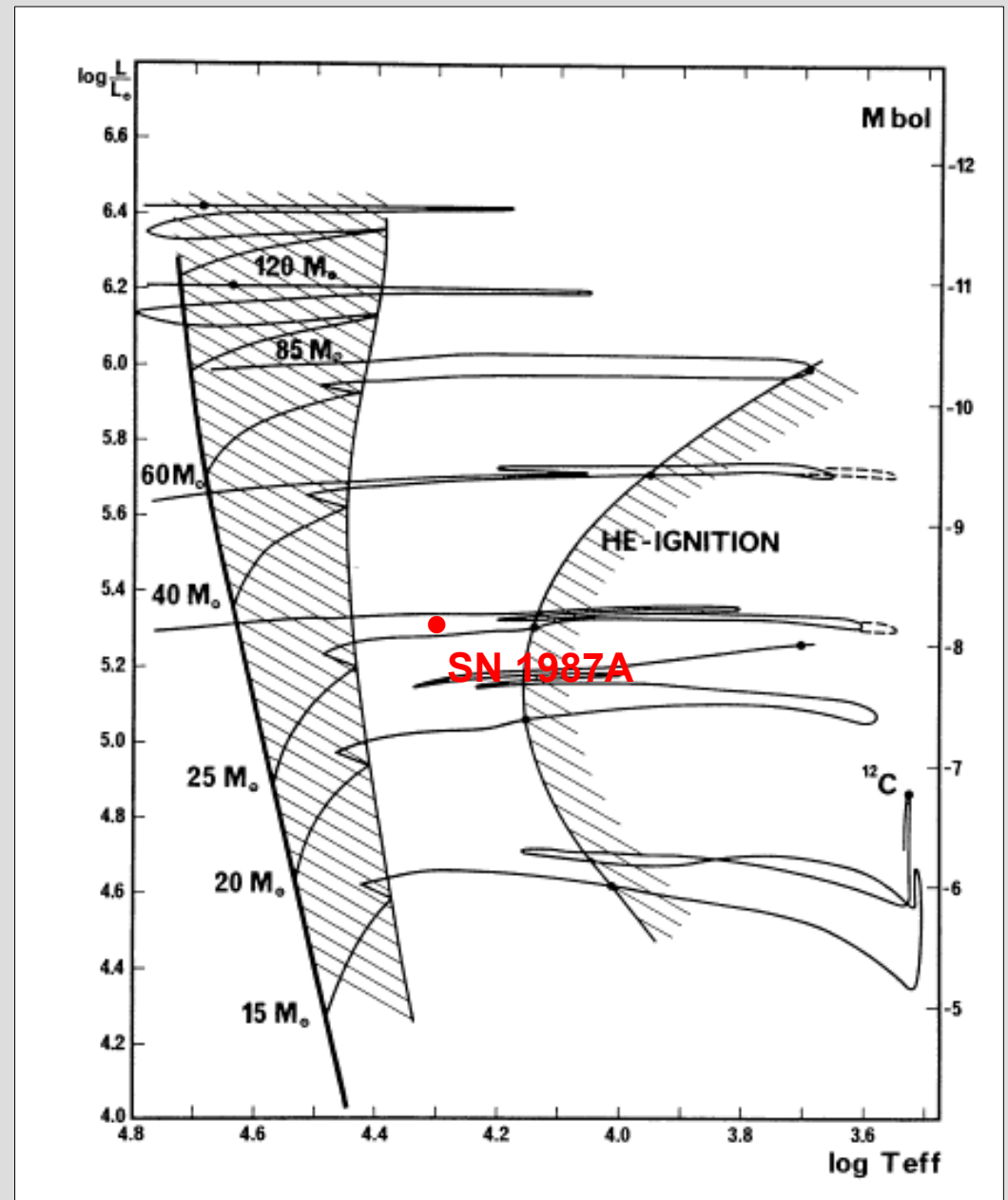
Progenitor star is Sk -69° 202  
and was **Blue Supergiant !!**



# Stellar evolution – models with mass loss !

Now in 1990

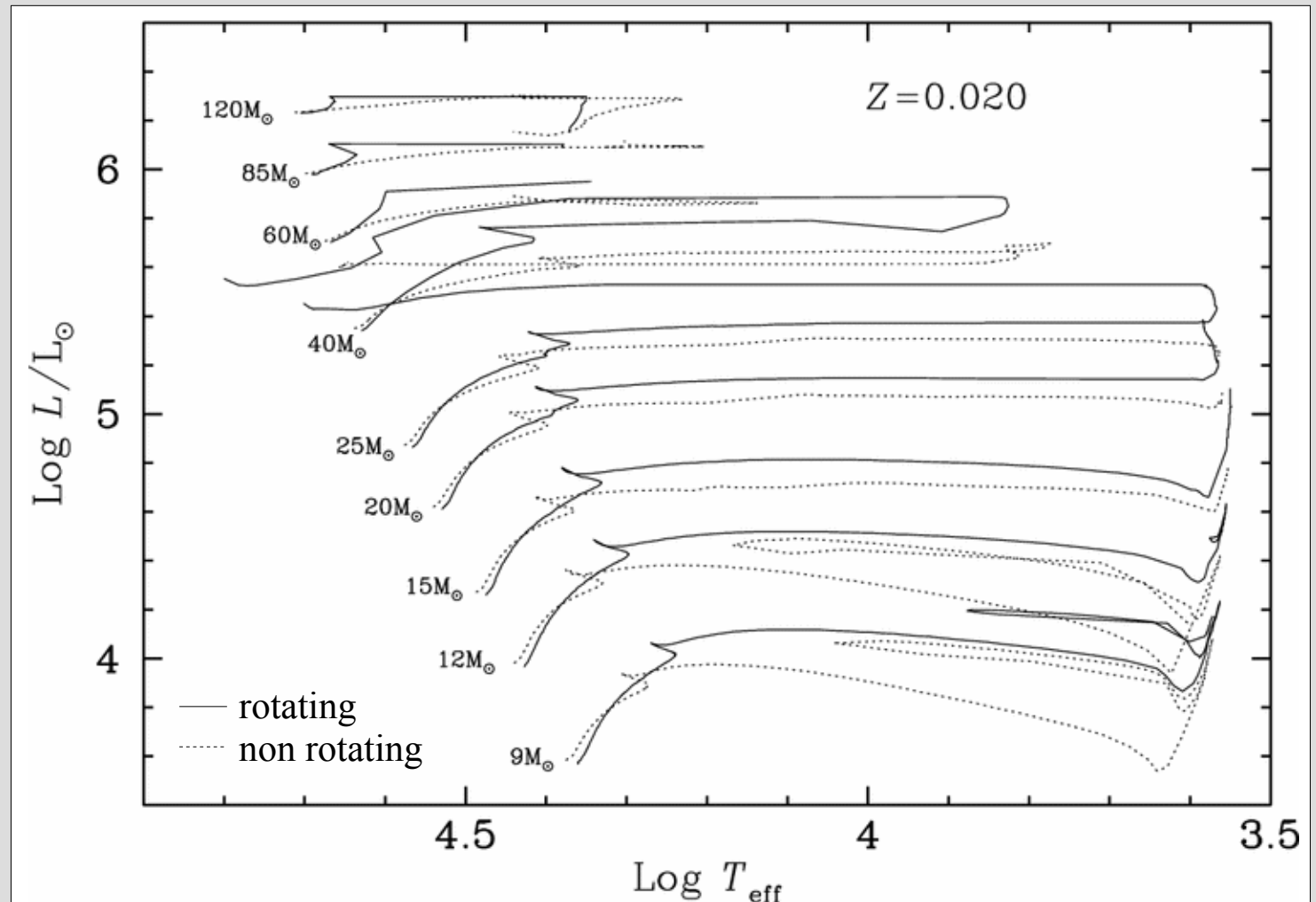
- added stellar winds  
**mass loss !**
- explained SN1987
- by including mass loss the models are able to explain a blue spuergiants as final phase before the supernova  
Since Stars change from the red to the blue.



# Stellar evolution – models with mass loss AND ROTATION

Starting in 2000

→ model now include **Mass loss** and **Rotation** !





# Stellar evolution – models with mass loss AND ROTATION

Starting in 2000

→ model now include **Mass loss** and **Rotation**

First idea and  
theoretical  
approaches  
already  
1955

→ no chance  
to caculate with  
the Computers  
at that time  
→ **need 3D**

Zeitschrift für Astrophysik, Bd. 38. S. 166–189 (1955)

Kleine Veröffentlichungen der Remeis-Sternwarte Bamberg Nr. 10

## Untersuchungen über rotierende Sterne

### I. Die Theorie nullter Ordnung

Von

**RUDOLF KIPPENHAHN**, Bamberg

Mit 1 Textabbildung

*(Eingegangen am 21. September 1955)*

Es wird die Theorie nullter Ordnung aufgestellt, die das Rotationsgesetz extrem langsam rotierender Sterne bestimmt. Es zeigt sich, daß ein Stern nach hinreichend langer Zeit einem Rotationsgesetz zustrebt, das zwar wegen des ständigen Drehimpulsverlustes durch Strahlung nicht stationär ist, das aber vom Anfangsrotationsgesetz nicht mehr abhängt. Die Bestimmung dieses Gesetzes und die Frage, wie ein Stern aus einem beliebigen Anfangszustand heraus dieses Rotationsgesetz erreicht, führen unter vereinfachenden Annahmen (spezielle Vorschriften für den

# Stellar evolution – models with mass loss AND ROTATION & MAGNETIC FIELDS !

Modells 2010+

→ model now include **Mass loss** and **Rotation** and **MAGNETIC FIELDS** !

## Stellar evolution with rotation and magnetic fields

### III. The interplay of circulation and dynamo

A. Maeder and G. Meynet

Geneva Observatory, 1290 Sauverny, Switzerland  
e-mail: [andre.maeder;georges.meynet]@obs.unige.ch

**Abstract.** We examine the effects of the magnetic field created by the Tayler-Spruit dynamo in differentially rotating stars. Magnetic fields of the order of a few  $10^4$  G are present through most of the stellar envelope, with the exception of the outer layers. The diffusion coefficient for the transport of angular momentum is very large and it imposes nearly solid body rotation during the MS phase. In turn, solid body rotation drives meridional circulation currents which are much faster than usual and leads to much larger diffusion coefficients than the magnetic diffusivity for the chemical species. The consequence is that the interplay of the thermal and magnetic instabilities favours the chemical transport of elements, while there would be no transport in models with magnetic field only. We also discuss the effects on the stellar interior, lifetimes and HR diagram.

**Key words.** stars: rotation – stars: magnetic fields – stars: evolution





# Stellar evolution – models with mass loss AND ROTATION & MAGNETIC FIELDS !

Modells 2010+

→ model now include **Mass loss** and **Rotation** and **MAGNETIC FIELDS** !

