8. One important cooling mechanism for gas in the interstellar medium is emission in the forbidden C II line at 157.7 μm. The line results from a so-called magnetic dipole tranisition between two atomic levels in the ground state of C II. The lower level has a statistical weight of 2 and the upper level has a statistical weight of 4. The Einstein coefficient for this transition is $A = 2.35 \times 10^{-6}$ s.

Suppose we observe a cloud and know that it is spherical, with a radius of 1 pc and a distance of 50 pc. Its temperature has been estimated to be 50 K, and its observed flux is 5×10^{-14} W m^{-2} (integrated over the entire line). Estimate the density of hydrogen atoms in the cloud.

Here you can assume that all C^+ ions are found in the ground state, and that the ground state is composed only of the two levels mentioned above. Assume also that cosmic radiation has ionized the neutral carbon so that the ionization fraction is

$$\frac{n\,(\mathrm{C\,II})}{n\,(\mathrm{C\,I})} \quad = \quad 10^{-4}$$

You can also assume nearly solar metallicity, i.e., $\log \epsilon(C) = 8.6$ on the scale on which $\log \epsilon(H) = 12.00$.

Hint: For the calculations, first assume that the level populations in the ground state are Boltzmann-distributed. Verify this afterwards by noting that for this transition,

$$Q_{12} \equiv \langle v \cdot \sigma_{12} \rangle = 8 \times 10^{-16}\ \mathrm{m^3\,s^{-1}}$$

How will this estimate influence whether the Boltzmann distribution can be used or not?