Radiative Processes in Astrophysics
Problem Set 1

Due date: Wednesday, 13 March 2002

1. Maxwell distribution

(a) What is the Maxwell-Boltzmann distribution? Write down the formula and explain it. Use a computer program to plot the distribution for two or three different temperatures.

(b) What is the most probable speed, the average speed and the root-mean-square (r.m.s.) speed for particles of a given mass m and temperature T? How are these obtained from the Maxwell distribution?

(c) What are the r.m.s. speeds of the electrons, hydrogen atoms and iron atoms in the Sun's photosphere? Compare this to the r.m.s. speed of oxygen molecules in the Earth's atmosphere.
(Note: You will have to decide what temperature to use in each case.)

2. Ideal gas law

The center of a star contains 60% hydrogen, 35% helium and 5% metals. The density is $\rho = 50$ g cm^{-3} and the temperature is $T = 1.5 \times 10^7$ K.

(a) What is the mean molecular weight of this gas?
(Hint: You can assume that at this temperature, the gas is fully ionized. For the heavier elements, you can use the approximation that the atomic number is a bit less than half of the atomic weight: $Z_i + 1 \approx \frac{1}{2} A_i$.)

(b) Assuming an ideal gas, estimate the gas pressure.

3. Saha equation

Suppose you have to calculate the number densities of different ions (relative to that of HI) in different regions in a star. You have a table of temperatures T and electron pressures P_e (in cgs units) for the different regions, as well as a table of ionization energies χ (in eV) for the different species.

(a) How would you rewrite the Saha equation to make it easier to plug in the appropriate numbers in this case?

(b) Using the expression you derived in part (a) above, calculate the ratios $N(\mathrm{H}) \,/\, N(\mathrm{H}^-)$ and $N(\mathrm{H\,II}) \,/\, N(\mathrm{H})$ for a typical region in the the Sun's atmosphere with e.g., $T = 6428$ K, $\log P_e = 1.80$.

Note: The ionization energy for H^- is $\chi = 0.754$ eV. Explain why the partition functions of H^- and H II are equal to 1, and why the partition function of H I is very nearly 2.

4. Saha equation and Boltzmann distribution

How much of all magnesium can be found in the excited 3d ^{2}D state of Mg II in a gas with a temperature of $T = 5000$ K and the following electron densities?

(a) $n_e = 10^{10}$ cm^{-3}

(b) $n_e = 10^{15}$ cm^{-3}

The attached photocopies, taken from reference books in the library here, contain some of the information you need to solve this problem.