Radiative Processes in Astrophysics Problem Set 2

Due date: Wednesday, 20 March 2002

1. Radiation quantities

- (a) Define the following monochromatic quantities in terms of dE_{ν} , $d\nu$, dt, direction of the beam $\hat{\ell}$, $d\Omega$, and area $d\vec{A} = \hat{n}dA$. Explain what the various quantities are (include diagrams).
 - intensity I_{ν}
 - mean intensity J_{ν}
 - flux \mathcal{F}_{ν}
- (b) Why is it generally more useful to know I_{ν} than \mathcal{F}_{ν} ?
- (c) For which kind of astronomical objects is it possible to measure I_{ν} directly?
- (d) Does $\mathcal{F}_{\nu} = 0$ mean that there is no radiation?

2. Flux and Intensity

- (a) Assume two detectors are standing at the same distance from a point source of radiation. What is the ratio $\mathcal{F}_2/\mathcal{F}_1$ of the measured fluxes if the detector areas are $A_1 = 10 \, \mathrm{cm}^2$ and $A_2 = 30 \, \mathrm{cm}^2$?
- (b) Now assume there is a third detector which is twice as far away from the point source and has an area of $A_3 = 10 \,\mathrm{cm}^2$. What is the ratio $\mathcal{F}_3/\mathcal{F}_1$?

3. Conversions

- (a) Show that $d\nu/\nu = -d\lambda/\lambda$.
- (b) Are I_{ν} and I_{λ} equal for a given beam? (E.g., show that $\nu I_{\nu} = \lambda I_{\lambda}$.)
- (c) Derive the expression for $B_{\lambda}(T)$ from

$$B_
u(T) = rac{2h
u^3}{c^2}\,rac{1}{\exp(h
u/kT)-1}$$

4. Radiation pressure

Use the equation for radiation pressure, together with the Stefan-Boltzmann law, to show that

$$P_{rad} = \frac{1}{3}aT^4$$

where $a = 4\sigma/c$. Write down the assumption(s) it was necessary to make along the way!