Radiative Processes in Astrophysics
Problem Set 3

Due date: Wednesday, 27 March 2002

1. Optical thickness and optical depth

(a) What is the difference between optical thickness and optical depth? Which of these is used more often for astronomical observations?

(b) Use the table below to calculate the geometrical depth ($z = -h$) at which we reach an optical depth of $\tau_5 = 1$ in the Sun.

T	$\log P_g$	$\log P_e$	h	$\log \rho$	$\log \kappa_5$	$\log N$
4400	3.7	−0.22	0	−7.7	−1.35	15.9
4450	3.97	−0.06	−40	−7.48	−1.22	16.17
4600	4.14	0.09	−80	−7.31	−1.10	16.34
4850	4.37	0.33	−125	−7.11	−0.92	16.54
5090	4.53	0.55	−160	−6.97	−0.78	16.68
5380	4.71	0.80	−200	−6.81	−0.61	16.84
5900	4.88	1.22	−260	−6.68	−0.34	16.97
6360	5.02	1.70	−300	−6.57	−0.04	17.08
7000	5.1	2.23	−320	−6.5	0.35	17.1
8300	5.2	3.05	−360	−6.4	1.00	17.2
9700	5.4	3.4	−380	−6.3	1.4	17.3

The table gives temperature T in degrees Kelvin, gas and electron pressure P_g and P_e in dyn cm^{-2}, height h in km, density ρ in g cm^{-3}, the mass absorption coefficient κ_5 at 5000 Å in cm^2 g^{-1}, and the number density of atoms (neutral or ionized) in cm^{-3}. Heights are measured at the base of the chromosphere, at which $\tau_5 = 0.004$.

2. Radiative transport equation

Write the differential form of the transport equation in terms of the emission and linear extinction coefficients. Then rewrite it by introducing the optical thickness and the source function. Finally, derive the formal solution (i.e., the integral form) by integrating both sides.

Hint: In order to perform the integration, first multiply both sides of the equation by a factor of e^{τ_ν}. What is the derivative of $I_\nu \, e^{\tau_\nu}$ with respect to τ_ν?

3. Radiation through a slab

An slab of material of thickness D is irradiated from one side with a beam of intensity $I_\nu(0)$. What is the emergent intensity $I_\nu(D)$?

(a) Assume pure emission:
the emission coefficient in the slab is j_ν and there is no extinction.

(b) Assume pure extinction:
the extinction coefficient in the slab is α_ν and there is no local emission within the slab.

(c) What are the results of (a) and (b) for a homogeneous slab?

(d) Why is it unrealistic to assume pure emission or pure extinction, especially for a thick slab?

(e) What is the emergent intensity for an optically thick (homogeneous) slab? How do you interpret this result?

4. Total source function

(a) Suppose there are two physical processes A and B with corresponding extinction coefficients α_ν^A and α_ν^B. Both processes contribute to the extinction of a beam. How would you combine them to give the total extinction coefficient α_ν^{total}?

Do the same for the extinction coefficients κ_ν and σ_ν.

(b) The processes A and B each contribute to the emission as well. Compute the total emission coefficient j_ν^{total}.

(c) What are the source functions S_ν^A and S_ν^B for these processes? Write the total source function S_ν^{total} in terms of these source functions for the individual processes.