Stellar Structure and Evolution, Ht 2003
Extra problem 1

Due date: 11 September 2003 at 13:15

Marvin is a new graduate student who has just become the teaching assistant of a course on Stellar Structure and Evolution. The professor has asked Marvin to come up with a challenging problem for the exercises. It should help get the students to practice:

- looking up atomic data such as partition functions on their own
- thinking about mean molecular weight
- practice using the Saha equation

Below you can read the question that Marvin has invented, as well as his solution for the answer key. Unfortunately, there is one very fundamental flaw in Marvin's question. Can you find what it is? In what ways could he rework the problem to make it usable and educational for the students to solve?

Question:

Consider a gas that contains 90% hydrogen and 10% titanium (by mass) at a temperature of $T = 4000$ K. The density of this gas is $\rho = 2.21 \times 10^{-7}$ g cm^{-3}. If exactly half of the hydrogen atoms are ionized, what is the gas pressure?

Answer:

Roughly half of the titanium is doubly ionized and the other half is singly ionized, so for the mean molecular weight μ we have:

$$\frac{1}{\mu} = \frac{1}{1} \cdot \left(\frac{1}{2} X_H\right) + \frac{2}{1} \cdot \left(\frac{1}{2} X_H\right) + \frac{2}{22} \cdot \left(\frac{1}{2} X_{Ti}\right) + \frac{3}{22} \cdot \left(\frac{1}{2} X_{Ti}\right) \quad (1)$$

$$\frac{1}{\mu} = \frac{3}{2} X_H + \frac{5}{44} X_{Ti} \quad (2)$$

$$\frac{1}{\mu} = \frac{3}{2}(0.90) + \frac{5}{44}(0.10) = 1.36 \quad (3)$$

$$\mu = 0.7345576 \quad (4)$$

Assuming an ideal gas, the corresponding gas pressure is

$$P_g = nkT = \frac{\rho}{\mu m_u} kT \quad (5)$$

$$P_g = 10^5 \text{ dyn cm}^2 \quad (6)$$