Stellar Structure & Evolution
Old Exam Questions, Ht2003

1. Derive the equation of hydrostatic equilibrium.

2. Consider a star in radiative equilibrium

$$L \propto \frac{T^3}{\kappa\rho}\frac{dT}{dr}R^2$$

   where κ is the opacity in units of m^2 kg. Use the equation of hydrostatic equilibrium and the equation of state for an ideal gas to show that this will lead to an approximate mass-luminosity relation for main-sequence stars. Assume that κ and the mean molecular weight are constant.

3. (a) Make a 5×3 table which names the major energy-transport mechanisms (i) close to the center, (ii) "half-way" in mass, i.e., at a radius corresponding to $m(r) = 0.5M_\odot$, and (iii) just below the photosphere. Do this for main-sequence stars with masses (A) $0.1M_\odot$, (B) $1M_\odot$, (C) $2M_\odot$, (D) $100M_\odot$ and (E) in a white dwarf star of $0.5M_\odot$.

   (b) Describe the transport mechanisms and under which conditions they dominate.

4. Which is the most important opacity source in

   (a) the Sun?

   (b) a $5M_\odot$ dwarf?

   (c) a $15M_\odot$ main-sequence star?

5. (a) Derive the expressions for the mean molecular weight in the two cases of fully neutral and fully ionized matter. Use the mass fractions X, Y and Z to define the chemical composition.

   (b) Calculate the mean molecular weight at the center of the Sun when it was on the ZAMS, with $X = 0.71$ and $Y = 0.27$, and compare it to its value at the present time, at which $X = 0.34$ and $Y = 0.64$. (For simplicity, you may assume that there are no metals: $X = 0.35$ and $Y = 0.65$.)

   (c) Explain *how* and *why* this change in the mean molecular weight has affected the Sun's luminosity during its time on the main sequence.

6. If convection in the Sun were to stop, how would its luminosity, effective temperature and radius be affected? Why?

7. What is meant by "homologous" stellar models? Why are such models discussed, despite the fact that real stars are not homologous?

8. What is the Hayashi track, and what is the characteristic feature of the structure of stars on the Hayashi track?

9. A hypothetical, homogeneous, optically thin interstellar cloud consisting of 100% H_2 gas has $\rho = 10^{-21}$ g cm^{-3} and $T = 300$ K. Its mass is equal to the Jeans mass, which may be estimated from

$$M_{\rm crit} \approx \frac{T^{3/2}}{\rho^{1/2}} \left(\frac{\pi k}{m_u G \mu} \right)^{3/2} \tag{1}$$

(where μ is a dimensionless number and m_u is an atomic mass unit).

(a) What is the density and temperature of a gas cloud with the same Jeans mass as above but which is on the line $t_h = t_c$ in Fig. 1?

(b) Assume that the original cloud quickly cools to the line $t_h = t_c$ under constant density (10^{-21}) and that it then fragments into a number of Jeans-mass clouds. How many fragments would then be formed?

(c) If the H_2 cloud instead cooled adiabatically (along the line $T \propto \rho^{\gamma-1}$) to the line $t_h = t_c$, what would then be the temperature, density and Jeans mass? Adopt the value for a perfect diatomic molecular gas $\gamma = C_P/C_V = 1.40$, which is close to the real value for H_2 ($\gamma = 1.41$).

(d) Which one of the 3 cooling scenarios above is most like the real case?

(e) What is special about gas at the line $t_h = t_c$?

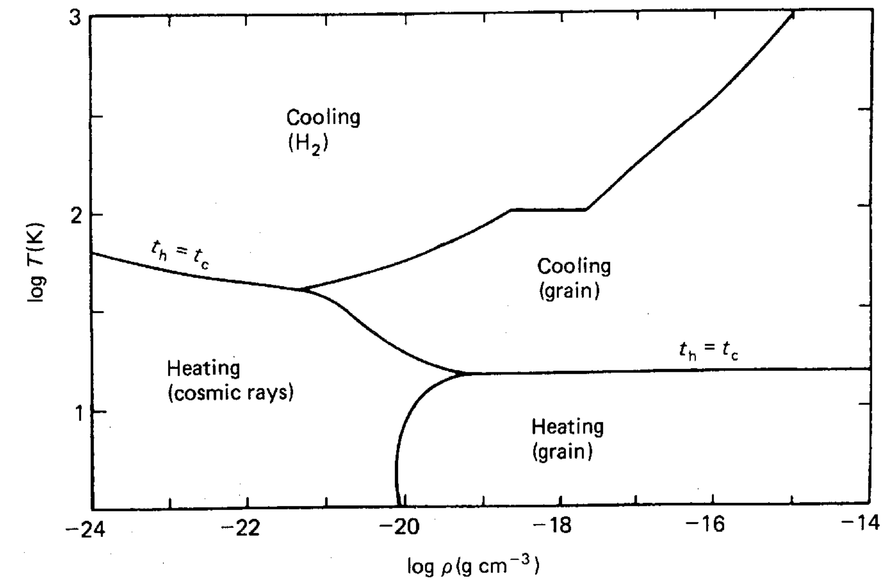

Figure 1: Domains in the log ρ, log T plane (Böhm-Vitense, E. 1992, *Introduction to stellar astrophysics, Vol. 3*, Cambridge University Press, Cambridge, p. 252. Adapted from C. Hayashi 1966, Ann. Rev. Astron. Astrophys. 4, 171.)

10. A star has a luminosity of 4×10^{29} W, $T_{\rm eff} = 3500$ K and a mean density of $\bar{\rho} = 2 \times 10^{-3}$ kg m^{-3} when core helium burning starts. The metallicity is 1/80 of the solar metallicity. Calculate its approximate mass and describe in detail the helium burning and the probable remaining evolution of the star for many bilions of years. Indicate roughly the amounts of time spent in the different stages.

11. Explain what the κ mechanism is and how it works.

12. What is the instability strip? Explain what determines its "red" and "blue" limits.

13. What relation between different quantities did Henrietta Leavitt discover for Cepheid variables? What is the most important application of this relation?

14. (a) Indicate the approximate positions of RR Lyrae stars and classical Cepheids in an H-R diagram, with effective temperature and luminosity on the axes.
    (b) What is characteristic for these stars? What process causes the phenomenon? How and where does it work?
    (c) What are the relevant time scales for each type of star?

15. Which fundamental force of nature releases the basic energy to power a supernova explosion of Type II?

16. Estimate the gravitational "energy production" for a core-collapse supernova. Assume that the collapsing iron core has a mass of $3M_\odot$ and the radius of a white dwarf, and that it collapses to a radius typical for a neutron star. Assume that both objects have constant density both before and after the collapse.

17. What is the determining difference between the optical spectra of Type I and Type II supernovae?

18. What is the (a) energy source and (b) atomic process that keeps a planetary nebula shining?

19. (a) What is the mass range for main-sequence stars that end up as white dwarfs?
    (b) What is the typical mass, density and radius of a white dwarf?
    (c) What is the upper mass limit for a white dwarf?
    (d) What is the energy source for a white dwarf? How is energy transported in a white dwarf?
    (e) What is the typical time-scale for the cooling of white dwarfs?

20. (a) What is the typical mass, density and radius of a neutron star?
    (b) What exactly keeps a neutron star from collapsing to a black hole?

21. What is meant by an "isochrone"?

22. Describe some kind of observations that are used for testing theories of stellar evolution and the idea behind the test.

23. "The first dredge-up" is often evidenced in the spectra of globular cluster Giant stars. Which elements and isotopes change relative to their main-sequence abundances and in which directions?

24. How does the isotope ratio $^{12}C/^{13}C$ change for a solar-type star between the main sequence and the end of the AGB phase? At exactly which evolutionary phase does this change occur, and which atomic process is responsible for the change? How can this been seen from observations? What allows us to see evidence that this is happening in the interior of the star?

25. In an HR diagram (or colour-magnitude diagram) of stars in the solar vicinity, the main sequence shows a width of about 1–1.5 magnitudes. Stars on the more luminous side have generally spent most of their core hydrogen. A small fraction of the stars stick out by showing up 0.5–1 magnitudes fainter than the typical lower limit for the disk stars.

    (a) What are these fainter stars called?

    (b) What is the main intrinsic physical difference between these and the "normal" stars?

    (c) In what other characteristics do they differ?

    (d) Which two partly counteracting effects cause the effect seen in the HR diagram which was mentioned in part (25a)?

26. (a) Draw a typical (theoretical) HR diagram of a globular cluster of our Galaxy. Label and indicate typical numbers along the axes.

    (b) What is the typical mass in $M_\odot$, effective temperature, and luminosity in $L_\odot$ for globular cluster stars in the turn-off region?

    (c) Roughly how does the present-day stellar mass vary with position along the whole sequence of stars that you have drawn?

    (d) What is the primary "chemical" difference between Milky Way globular cluster *dwarf* stars and the Sun? *Why* is there such an abundance difference?

    (e) The "first dredge-up" is often evidenced in the spectra of giant stars in globular clusters. Which elements and isotopes are observed to change their abundances when this occurs? How do these "new" abundances differ relative to the main-sequence abundances? (Note that the abundances of H and He are **not** reliably observable.)

    (f) What is the name and net effect of the nuclear process that it responsible for these abundance changes?

27. Study the H-R diagram for the globular cluster NGC 5272 in Fig. 2.

(a) What is the cluster's approximate distance from the Sun?

(b) About how old are the globular clusters that (like NGC 5272) belong to the Milky Way?

(c) What is the mass of the most massive main-sequence stars in the cluster? Using this value as a starting point, estimate the *current* mass of the stars at positions A–F in the diagram (in units of solar masses).

(d) What are the names for (i) the area below B, (ii) around C, (iii) the region between C and D? What do we call (iv) the stars around E and (v) the stars around and to the left of F?

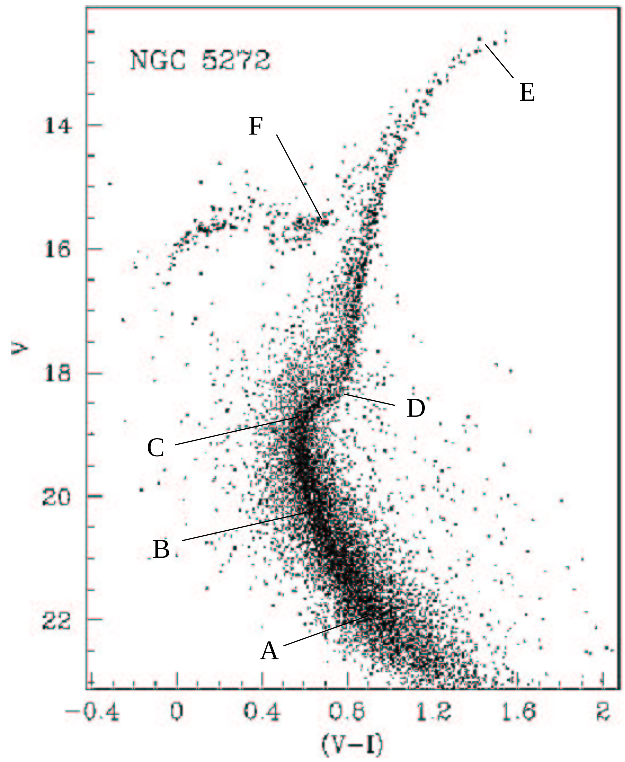

Figure 2: H-R diagram of globular cluster NGC 5272 (Rosenberg et al., Astron. Astrophys. Suppl. 145, 451).