Stellar Structure & Evolution
Problem Set 2, Ht2003

Due date: Tuesday, 16 September 2003 at 10:15

1. Stellar structure equations

Write down *and discuss* the basic equations of stellar structure. Note that either mass or radius can be used as the independent variable. When do you think it would be more convenient to use radius, and when mass?

Note also that a realistic stellar model might have, e.g., a radiative envelope and a convective core. For the temperature equation, consider the two extreme cases: radiative equilibrium and pure convection.

2. Pressure scale height

(a) Explain the physical meaning of the (isothermal) pressure scale height H_p, defined as

$$\frac{1}{H_p} \equiv -\frac{1}{P}\frac{dP}{dr} , \qquad (1)$$

(b) What is the typical scale height in the Earth's atmosphere (which contains N_2 and O_2 molecules and has oxygen and nitrogen mass fractions $X_O \approx 0.2$ and $X_N \approx 0.8$, respectively)? Compare this with the scale height in the Sun's atmosphere (mass fractions $X \approx 0.75$, $Y \approx 0.25$) and in a red supergiant.

3. Adiabatic temperature gradient

Where does the adiabatic temperature gradient ∇_{ad} come from? In this problem you will use the first law of thermodynamics and the perfect gas law to derive the adiabatic temperature gradient as well as the adiabatic gas law.

(a) Explain why, for a gas, the first law of thermodynamics takes the form

$$dQ = dU + P\,dV \qquad (2)$$

(b) Use the first law and the definition of the specific heat C_V to show that

$$C_V = \frac{dU}{dT} \qquad (3)$$

(c) Use the definition of the specific heat C_P and the perfect gas law

$$PV = \mathcal{N}\mathcal{R}T \qquad (4)$$

to show that

$$C_P = C_V + \mathcal{N}\mathcal{R} \qquad (5)$$

What is the physical reason for why C_P is always greater than C_V?

(d) Now apply the first law of thermodynamics to an adiabatic process, and use the perfect gas law to obtain the adiabatic temperature gradient

$$\nabla_{ad} = \left(\frac{d\ln T}{d\ln P}\right)_{ad} = \frac{\gamma - 1}{\gamma} \qquad (6)$$

where $\gamma = C_P/C_V$ is the ratio of specific heats.[1]
Discuss how γ can take on different values depending on the conditions in the gas.

(e) Calculations done in part (d) can be integrated to obtain a relation between P and T that leads to the adiabatic gas law

$$P = K\rho^{\gamma} \qquad (8)$$

where K is a constant. Show how this is done.

4. Convection

(a) What are *convection* and *convective instability*? What is the *Schwarzschild criterion*?

(b) Explain how convection zones in stars are affected by ionization, effective temperature and chemical composition.

5. Student input

(a) What point(s) in the book or lecture were not very clear? What topic(s) would you like to hear/talk more about in the tutorial?

(b) If you were the teacher, what would you ask about the material covered so far? Try to make up a sample problem plus a solution, or at least two simpler questions and their answers. You might want to choose something you mentioned in part (a), or simply something you found interesting.

[1] Hint: The perfect gas law can be differentiated with respect to T and used to eliminate V from the equation

$$\frac{dU}{dT} + P\frac{dV}{dT} = 0 \qquad (7)$$