Stellar Structure & Evolution
Problem Set 3, Ht2003

Due date: Thursday, 25 September 2003 at 10:15

1. **Nuclear fusion**

   (a) Estimate the average kinetic energy of the nuclei in the core of a typical hydrogen-burning star. Compare this with the height of the Coulomb energy barrier at the surface of a nucleus, where $\mathcal{Z}_1$ and $\mathcal{Z}_2$ are the atomic numbers of the nuclei taking part in the reaction. Give your answers in terms of both eV and temperature.

   (b) Why are nuclear reactions are able to take place despite such large energy (or temperature) differences? Explain the Gamow peak, and draw it schematically *with useful labels* on the axes. What two considerations are responsible for its shape and position?

2. **Stellar fuels** (Prialnik, exercise 4.2)

   Consider a star made up of non-degenerate gas with solar composition ($\mu = 0.61$, $\mu_e \approx 1.17$) and a density profile of

$$\rho = \rho_c \left[1 - \left(\frac{r}{R}\right)^2\right] \tag{1}$$

   Using the threshold temperatures in the table below, estimate the minimum stellar mass required for the central ignition of the different nuclear fuels.[1]

Fuel	Process	Products	$T_{\rm min}$ (K)	$M_{\rm min}$ (kg)	$M_{\rm min}$ ($M_\odot$)
H	p–p		$\sim 4 \times 10^6$		
H	CNO		1.5×10^7		
He	3α		1×10^8		
C	C + C		6×10^8		
O	O + O		1×10^9		
Si	Nuclear equil.		3×10^9		

[1] *Hint:* First write the central density ρ_c, pressure P_c and temperature T_c in terms of stellar mass M and radius R; use these relations to get ρ_c in terms of T_c and M. The requirement that the electron gas pressure P_e must be higher than the (non-relativistic) electron degeneracy pressure $P_{\rm deg} = 10^7(\rho/\mu_e)^{5/3}$ (SI units) yields the necessary inequality relating stellar mass and central temperature.

3. **Old exam questions**

   (a) Describe in detail the two processes which supply almost all the energy to stars on the main sequence. At which main-sequence masses does each process dominate?

   (b) How do we directly observe the nuclear processes occuring in the solar center?

   (c) Which are the two most important reasons why a solar-type star will burn hydrogen in the core for up to ten times longer than it will later burn helium in the core?

   (d) What important fusion processes take place in stars after they leave the main sequence? Which are the most important reactions in each of those processes?

   (e) r and s processes

      i. What is meant by the r process and the s process? What is the specific ("absolute") reason for their names?

      ii. What is the important outcome of each of these processes? How are the results different?

      iii. In which type of stars (evolutionary stages) is each process thought to occur?

4. **Energy generation rates**

   (a) The energy generation rate is often approximated by a power law of the form

   $$q \;=\; \text{const.}\ X_i X_x\, \rho^\alpha\, T^n \tag{2}$$

   Give the appropriate expressions for the pp chain, the CNO cycle and the triple-alpha process. Explain the reason for each value of the exponent α! Why do different books give different values for the exponent n?

   (b) How much of the energy produced in the center of the present-day Sun comes from the pp chain, the CNO cycle and the triple-alpha process?[2]

[2] *Hint:* Assume $\rho = 162$ g cm^{-3}, $X = 0.34$, $X_{\mathrm{CNO}} \approx 0.013$. Formulas for the energy generation rate can be found, e.g., in Carrol & Ostlie and in Kippenhahn & Weigert.