Stellar Structure & Evolution, Fall 2002
Problem Set 1

Due date: Thursday, 12 September 2002

1. Virial theorem

(a) Show that the gravitational potential energy of a star is

$$\Omega = -\int_0^M \frac{Gm\,dm}{r} = -3\int_0^R \frac{P}{\rho}\,dm \qquad (1)$$

(b) Derive the virial theorem in the form $\Omega = -2\,U_{\rm int}$, where $U_{\rm int}$ is the total internal, or thermal, energy of the star. Note what assumptions you made! Use this result to explain why half of the gravitational potential energy released in contraction goes into heating the stellar gas, and half gets radiated away. If energy were to get pumped into the star, how would it get divided up between thermal and potential energy?

(c) What is the virial theorem for radiation? What does this tell you about the stability of a star dominated by radiation pressure?

2. Stellar timescales

Changes in a star occur on different timescales, which can be estimated in a variety of ways. Derive equations for the timescales below, mentioning the important assumptions/approximations you made along the way. Express your answers in terms of the solar mass $M_\odot$, radius $R_\odot$ and luminosity $L_\odot$.

Once you have done that, you can also get a feeling for how long these timescales are for different stars, which range (very roughly!) in mass from $0.1M_\odot$ to $100M_\odot$, in radius from $0.01R_\odot$ to $1000R_\odot$, and in effective temperature from 3000 K to 50,000 K. What are the corresponding ranges for the different timescales?

(a) **The dynamical timescale**
How long does it take for a test particle to fall a certain distance in the gravitational potential well of a star of mass M and radius R? How long does it take for a sound wave to travel a certain distance in the star? (What characteristic distance did you use for your estimate, and why?)

(b) **The Kelvin-Helmholtz timescale**
About how long would it take for a star to radiate away all of its gravitational potential energy by contraction? Why is this timescale for the release of gravitational energy also known as the Kelvin-Helmholtz, or "thermal" timescale?

(c) **The nuclear timescale**
Roughly how long can a star's luminosity be maintained from the energy released by nuclear fusion in its core?

3. Pressure scale height

(a) Explain why the quantity H_p, defined as

$$\frac{1}{H_p} \equiv -\frac{1}{P}\frac{dP}{dr}\,, \tag{2}$$

is called the (isothermal) pressure scale height.

(b) Compare the typical pressure scale height in the Earth's atmosphere (which contains N_2 and O_2 molecules and has oxygen and nitrogen mass fractions $X_O \approx 0.2$ and $X_N \approx 0.8$, respectively) with that in the Sun's atmosphere (mass fractions $X \approx 0.75$, $Y \approx 0.25$).

4. Adiabatic temperature gradient

Use the first law of thermodynamics to derive the adiabatic gas law. Show that if $\gamma = C_P/C_V$ is the ratio of specific heats, then the equation of state for an ideal gas undergoing an adiabatic process is

$$PV^\gamma = K \tag{3}$$

where K is a constant. Use this to derive the adiabatic temperature gradient

$$\nabla_{ad} = \left(\frac{d\,\ln T}{d\,\ln P}\right)_{ad} = \frac{\gamma - 1}{\gamma} \tag{4}$$