Stellar Structure & Evolution, Fall 2002
Problem Set 2

Due date: Thursday, 3 October 2002

1. Radiative temperature gradient

Derive the temperature equation

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\rho\kappa}{T^3}\frac{L(r)}{4\pi r^2} \tag{1}$$

directly from the quation of radiative transfer equation! As always, note the assumptions made at each step.

Hint: Begin with

$$-\frac{\cos\theta}{\rho\kappa_\nu}\frac{dI_\nu}{dr} = I_\nu - S_\nu\,, \tag{2}$$

where θ is the angle between the light ray and the outward radial direction. Multiply this by $\cos\theta$ and integrate over all angles and frequencies. Introduce the Rosseland mean opacity κ, which is defined via the relation

$$\frac{1}{\kappa}\int_0^\infty \frac{\partial B_\nu}{\partial T}\,d\nu \equiv \int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B_\nu}{\partial T}\,d\nu \tag{3}$$

2. Nuclear physics cross sections

Suppose we know the number density n_x of target particles and the number density n_i of incoming particles, as well as the relative speed v_i.

(a) Explain what the cross section $\sigma(E)$ for a given reaction means, in terms of the reaction rate and the flow of incoming particles. Then discuss why the reaction rate per unit volume is

$$r_{ix} = n_i n_x \langle\sigma v\rangle \left(\frac{1}{\delta_{ix}+1}\right) \tag{4}$$

where δ_{ix} is the Kronecker delta function,

$$\langle\sigma v\rangle = \int_0^\infty \sigma(v)\, v\, f(v)\, dv \tag{5}$$

and $f(v)$ is the distribution function for the speeds.

(b) Now for the Maxwell distribution,

$$f(v)\,dv = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) 4\pi v^2 dv \tag{6}$$

What is the physical meaning of $f(v)\,dv$? Show that this is equivalent to

$$\phi(E)\,dE = \frac{2}{\sqrt{\pi}}(kT)^{-3/2}\,E^{1/2}\exp\left(-\frac{E}{kT}\right)\,dE \tag{7}$$

Now rewrite Eq. (4) for r_{ix} as a proportionality in terms of the energy dependence (i.e., don't worry about the constants).

(c) Astrophysicists often write the cross section in the form

$$\sigma(E) = \frac{S(E)}{E} \exp\left(-\frac{b}{\sqrt{E}}\right) \tag{8}$$

where E is the (non-relativistic) kinetic energy of the particles in the center-of-mass system and $S(E)$ is a slowly varying function, at least for nonresonant reactions. At which energy E_0 does the reaction rate have its maximum?

(d) George Gamow showed that the probability for tunneling through the Coulomb barrier was proportional to

$$\exp\left(-\frac{4\pi^2 e^2 Z_1 Z_2}{hv}\right) \tag{9}$$

Use this to evaluate b in the expression for E_0. (Remember that v and E are defined in the center-of-mass system, so use the reduced mass.)

(e) And finally...estimate what ratios of E_0 to $\frac{3}{2}kT$ are typical for the pp chain and the CNO cycle!

3. Temperature dependence of nuclear energy generation

(a) Now that you have gone through the more complicated expressions obtained from nuclear physics, consider to the simpler form

$$\epsilon = \epsilon_0 X_i X_x \rho^\alpha T^\beta \tag{10}$$

Write down the appropriate expressions for the pp chain, the CNO cycle and the triple-alpha process. How is Eq. (10) related to the reaction rate and what approximations were made in order to obtain this simpler form? Discuss the different dependencies.

(b) Estimate how much of the energy production rate in the center of the Sun comes from the pp chain, the CNO cycle and the triple-alpha process.

4. Stability of stars

This question is meant to help you summarize, for yourself, chapter 6 in the book by Prialnik.

(a) What is meant by thermal, dynamical and convective (in)stability?

(b) Try to summarize what factors determine whether a star is stable in each of these senses. Discuss qualitatively the criteria for each type of stability. Then try to express these criteria in terms of the exponents α and β in the equation of state

$$P \propto \rho^\alpha T^\delta \tag{11}$$